Zhu Liping, Li Hongqi*, Wang Fenghui, Lv Jie, Sikandar Ali and Zhang Hong

A Flame Detection Method Based on Novel Gradient Features

Abstract: In this study, we present a novel approach to efficiently detect the flame in multiple scenes in an image. The method uses a set of parametric representation named as Gradient Features (GF), to learn the features of flame color changes in the image. Different from the traditional color features of the flame, GF represents the color changes in RGB channels for further consideration. In this study, support vector machine was applied to generate a set of candidate regions and the decision tree model was used to judge flame regions based on GF. Some exclusive experiments were conducted to verify the validity and effectiveness of the proposed method. The results showed that the proposed method can accurately differentiate between yellow color light and sunrise scenes. A comparison with the state-of-the-art preceding methods showed that this method can utilize the symmetry of flame regions and achieve a better result.

Keywords: Flame detection, SVM, decision tree, PCA, gradient features.

Introduction

Flame detection has drawn increasing attention from the research community, owing to its numerous potential applications in public safety affairs in the form of video analytics and others. The major task of flame detection is to detect the flame from static pictures in order to cope with fire accidents in the real world. Compared to normal object detection tasks, it is substantially more challenging, as a flame has no certain shape.

Over the past several years, advances in machine learning have led to remarkable progress in object detection and recognition. These existing approaches consist of two main parts: one part is used to obtain the candidate region of a flame based on some color model, while the other part is used to check whether the candidate region is a flame region or not by using some features such as edge feature [10], color feature [2, 6, 12, 13, 15], movement feature [6], and area feature [4]. Some commonly used models incorporate the background subtraction algorithm, Markov model, statistical analysis, cumulative matrix, and frequency analysis. According to the literature [1, 2, 4, 6, 12–15, 17], the accuracy of flame detection has been significantly improved recently. Nevertheless, the performances of flame detection methods remain unsatisfactory.

In this paper, an efficient method for flame detection with state-of-the-art accuracy in complex environments is proposed. The proposed method firstly nominates the candidate region of the flame based on color features, by checking if the pixel color is a flame color or not. Moreover, the statistical data show that there is a big difference between the flame region and the non-flame region in the changing of pixel values. Then, the Gradient Features (GF) representation is extracted to represent these changes. Specifically, the GF here is the mean variation of color gradients in different channels, and some statistical features based on the gradients are extracted to represent these changes. Furthermore, principal component analysis (PCA) is applied

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to decrease the feature dimension, and a decision model is trained to decide whether the region is a flame region or not. The experiments demonstrate that these representations of detection can achieve high-quality results in flame detection. The major contribution of this work is the development of a framework based on GF, a novel feature representation method for detecting flame in pictures.

Related Works

The typical approaches to flame detection consist of a probabilistic model, a background subtraction model, and a Markov model [1, 2, 14, 17], in which edge, color, and shape are the most common features, respectively.

Gomes et al. [6] applied a method to track a moving object and classify it based on the result of decreasing false ratio. The work of Toreyin et al. [13] was based on the dynamic background subtraction model, which is a fast-moving object detection algorithm. However, this kind of method depends solely on the extraction of the background image and is sensitive to obtrusive objects, such as floating leaves and moving red objects. Therefore, the dynamic feature of the flame is shaking instead of translating the whole object.

Dukuzumuremyi et al. [4] employed a method called stereo-based object detection to extract the foreground object and applied the wavelet transform method to generate features. Moreover, fuzzy-neural networks were used to judge whether the candidate region is a flame region or not.

Borges and Izquierdo [1] assumed that the pixel values of RGB channels follow an independent Gaussian distribution, and they compared the possibility of a flame to the highest possibility (peak of the histogram) of Gaussian distribution and tried to decide whether the region is a flame region or not. Zhang et al. [17] modified Borges and Izquierdo's method and considered the possibility of a flame region or not. In the second part of their method, they used changes in the area to judge the region of flame. However, these two methods are not convincible, as they assumed that the pixel value of RGB follows an independent Gaussian distribution. Obviously, compared to Gaussian distribution, multivariate normal distribution is more suitable in the real situation.

Lin et al. [9] also took advantage of color rules of RGB and hue intensity and saturation color space to quickly check a candidate flame region. However, due to difficulties in choosing a threshold of the rule, this method could not be applied in all situations.

Khatami et al. [8] used a combination of particle swarm optimization and the K-Medoids method to construct an optimum linear transformation matrix. They transformed the GRB color space to a new space, where the flame color is concentrated in a line, which makes it easier for the later judgment of the flame region.

Zhao et al. [18] applied a simple color rule and flame motion feature to roughly extract candidate flame regions and used the K-Singular Value Decomposition algorithm to achieve an overcomplete dictionary.

As the developments in deep learning methods increase, a recent work [16] incorporated deep neural networks into the detection frameworks and obtained improved performance. However, the deep learning method could not work when the region of flame is widely distributed in spatial space.

In this paper, we propose a statistical method named GF, which aims to capture the changes of color pixel in RGB channels. Exclusive experiments showed that our method outperforms the state-of-art techniques up to some extent.

Proposed Method

. Proposed System

The proposed framework takes re flame pixels in RGB space as input and the two-dimensional locations of the flame region in the image as output. Figure 1 illustrates the whole operational process.

As shown in Figure 1, the whole process consists of two main stages: the first stage is to obtain candidate flame regions by applying support vector machine (SVM) model-based labeled pixels to test the image, and the second stage is to judge the candidate region by applying the decision tree model that was trained on reduced GF.
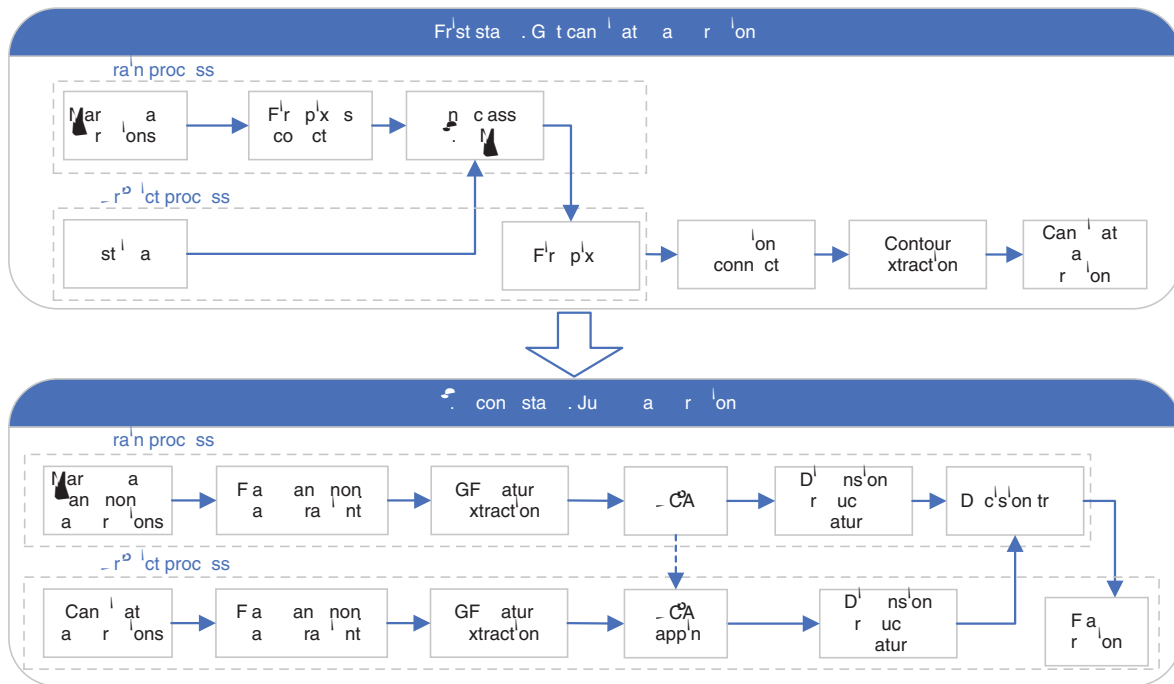
Figure 1: An Overview of the Structured Flame Detection Framework.

In the first stage of the framework, the SVM model is trained on manually collected fire flame pixels in RGB space. Then, the model is applied to find flame pixels in the image, and these flame pixels are used to build a connected region that will be marked as candidate flame regions. The candidate flame region is achieved by using a flood-fill algorithm based on the recognized flame pixels. Moreover, a decision tree model is trained to classify those candidate regions into two categories: flame region and non-flame region. In the next step, the GF is extracted from the candidate regions and some data pre-processing method such as PCA is applied to the extracted GF in order to reduce the feature dimension.

The second stage of the framework is used to judge whether the candidate region is a real flame region or a non-flame region. The real data show that there is a remarkable difference between the flame region and the non-flame region. Based on the candidate region of flame, some symmetric features such as changes of color values in eight directions are extracted. The mean and standard deviation of gradient values following all the eight directions of the region are computed. Specifically, 10 statistical values are extracted from the eight beams of fire, and the PCA method is applied to reduce the feature dimensions to six. Moreover, a decision tree model is then built on the reduced features to classify the region into a flame region or a non-flame region.

3 Candidate Flame Region Identification

To obtain candidate flame regions, our method traverses all pixels in an image and classify those pixels into flame or non-flame using one-class SVM. Indeed, a flood-fill algorithm is applied to organize flame pixels into a connected region named as candidate regions. Figure 2 illustrates the training process of this one-class SVM.
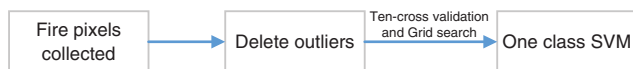

Figure 2: Training Process of SVM.

. .   Flame Color Features Extraction

There are many manual settings of ame features, such as contour features, motion features, and ame color features. Due to the irregularity of the ame contour, especially when facing blowing wind, judging ame by its contour is a di cult task. Human beings mostly use the color feature of the ame to distinguish ame in real life. As the distribution of a color value of ame pixels is very wide, this paper mainly focuses on red and yellow ames.

A widely used classi er, SVM [3], is applied for checking the category of a pixel roughly. Ho [7] applied SVM instead of the fuzzy logic threshold approach and obtained a good result. Di erent from Ho's work, we used one-class SVM to classify a pixel instead of two-class SVM in order to achieve better performance.

First, multiple ame images, as shown in Figure 3, were collected and the color of RGB channels of each ame pixel was extracted to construct the ame color space. Moreover, each ame pixel of the ame region was extracted and organized as a group, and the nally collected ame color space consisted of 95,619 samples and their distribution is shown in Figure 4.

As shown in Figure 4, the pixel value of the R channel was distributed more widely compared with the other two channels in ame images.
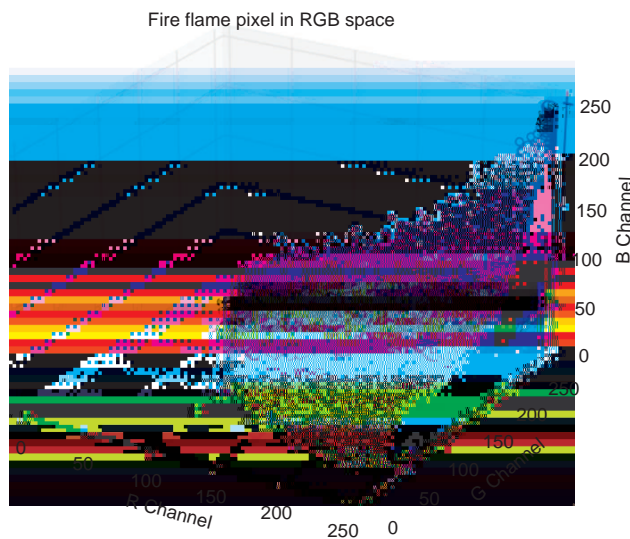

Figure 3: Marked Flame Pixels.

Figure 4: Distribution of Pixel Color Value in RGB Space.

Table 1: Parameters and Evaluation Results of Trained SVM Models.

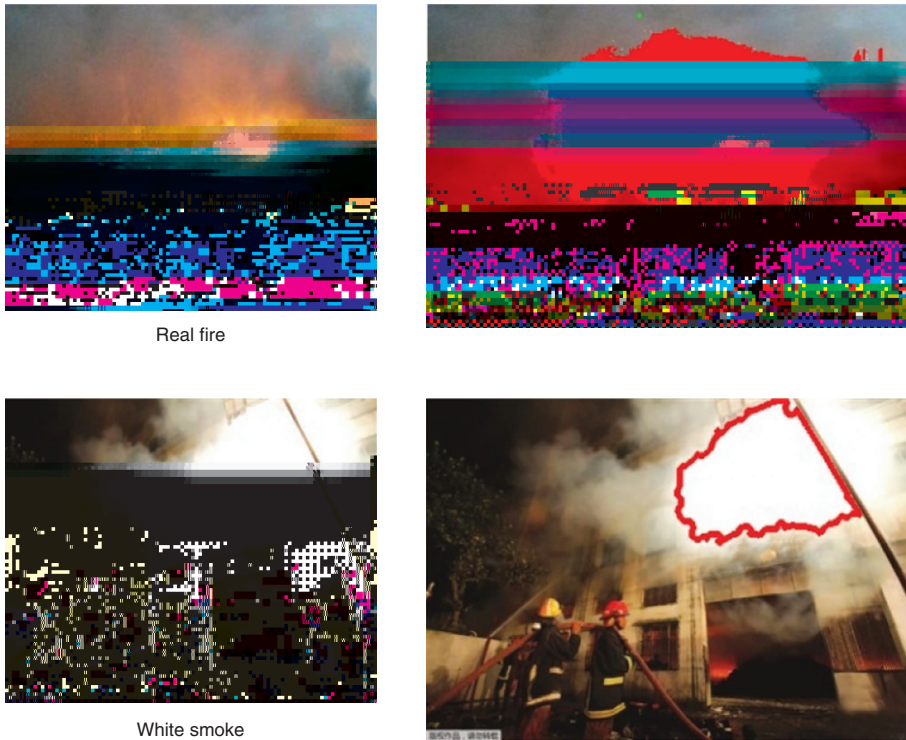|	Train Error	Test Error	Outlier Error	v	γ
RBF	.	.	.	.	.

Real fire

White smoke

Figure 5: Candidate Region Generation Based on the SVM Model.

. . Candidate Flame Region Generation

SVM is a widely used classi cation algorithm; it enables non-linear mapping into high-dimensional feature space without the associated computational cost based on the usage of the kernel method. In this study, a widely used kernel, the radial basis function (RBF) kernel, was applied in experiments. Moreover, a grid search method was utilized to help choose the optimum parameters. The chosen parameters of the SVM model are presented in Table 1.

The 10-cross validation method was applied to evaluate the models. When each pixel of the image was categorized into ame or no ame, the ood- ll algorithm was applied to organize these ame pixels into connected regions, as shown in Figure 5.

The trained SVM tries to nd the best possible candidate ame region. However, some ame-like regions are also viewed as ame regions due to its highly like ame in RGB color space. Some examples such as white smoke are shown in Figure 5. Therefore, it is necessary for further judgment of the candidate ame region to apply other models based on more representative features.

. Judgment of a Flame Region

In real life, judging a ame only by its pixel color is prone to errors, such as with moving maple leaves. Fly-ing maple leaves own the same color feature and moving feature as a ame, and therefore it adds to the recognition di culty of distinguishing it from the ame. Moreover, as the night light owns the same color

and symmetry region feature as the flame does, it is also difficult to differentiate it from real flame, if the background is ignored.

However, the change in pixel values is different between a flame and a non-flame object. Specifically, in flame regions, the red channel changes slowly, the blue channel changes very fast, and the changes of the green channel are in between.

Therefore, in this study, we took into consideration the pixel changes of flame and non-flame regions. The extracted features are supposed to represent the variation of pixel colors as in the real-world object and to obey a certain rule of nature. These rules state that the changes of pixel color value are a slow and gradual process. Moreover, the changing process of these pixel color values is different in the three channels.

The gradient computation for each pixel is shown in Figure 6.

As shown in Figure 6, there are eight adjacent pixels for the center pixel $P(x,y)$. We only computed the gradient values of the eight directions. One of the direct gradient computation is shown as a red arrow in Figure 6 and the formula is given below; it means the direction from $P(x,y)$ to $P(x-1, y+1)$.

$$G = \text{pixel}_{(x,y)} - \text{pixel}_{(x-1, y+1)}.$$

As a result, following the idea of pixel changes in the three channels, the gradient pixel color features, named GF, are designed to check whether a candidate region is a flame region or not. With the aim to obtain GF for the candidate flame region, we computed GF in eight directions instead of the whole region, as shown in Figure 7.

There are two reasons for computing the gradient in eight directions: to reduce the computing complexity and to obtain more representative gradient values as the flame region is a radially distributed region.

P(x−1, y+1)	P(x, y+1)	P(x+1, y+1)
P(x−1, y)	P(x, y)	P(x+1, y)
P(x−1, y−1)	P(x, y−1)	P(x+1, y−1)

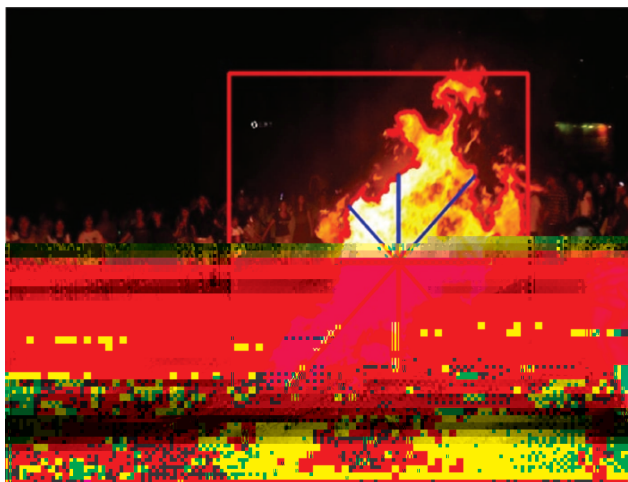**Figure 6:** Pixel Value of the Image in Spatial Space.

Figure 7: Region in the Red Frame is a Candidate Region.

Figure 8 illustrates the changes of pixel value following one direction for ame images and ame-like images in RGB color spaces.

As shown in Figure 8, the changes of pixel value in the R channel are much slower compared with the other two channels. This rule cannot be found in red clothes or white smoke, which can be mistaken for ame in the color feature space. The di erence in the pixel value changing rate in the three channels could be represented as a ratio of the average gradient of the R channel to the average gradient of the G channel.

Figure 9 demonstrates the distribution range of the ame and non- ame regions.

Figure 9 shows the gradient range of real re, red clothes, and white smoke. It is obvious that the gradient range of real re is much wider compared with other ame-like objects. From Figure 9, we can see that
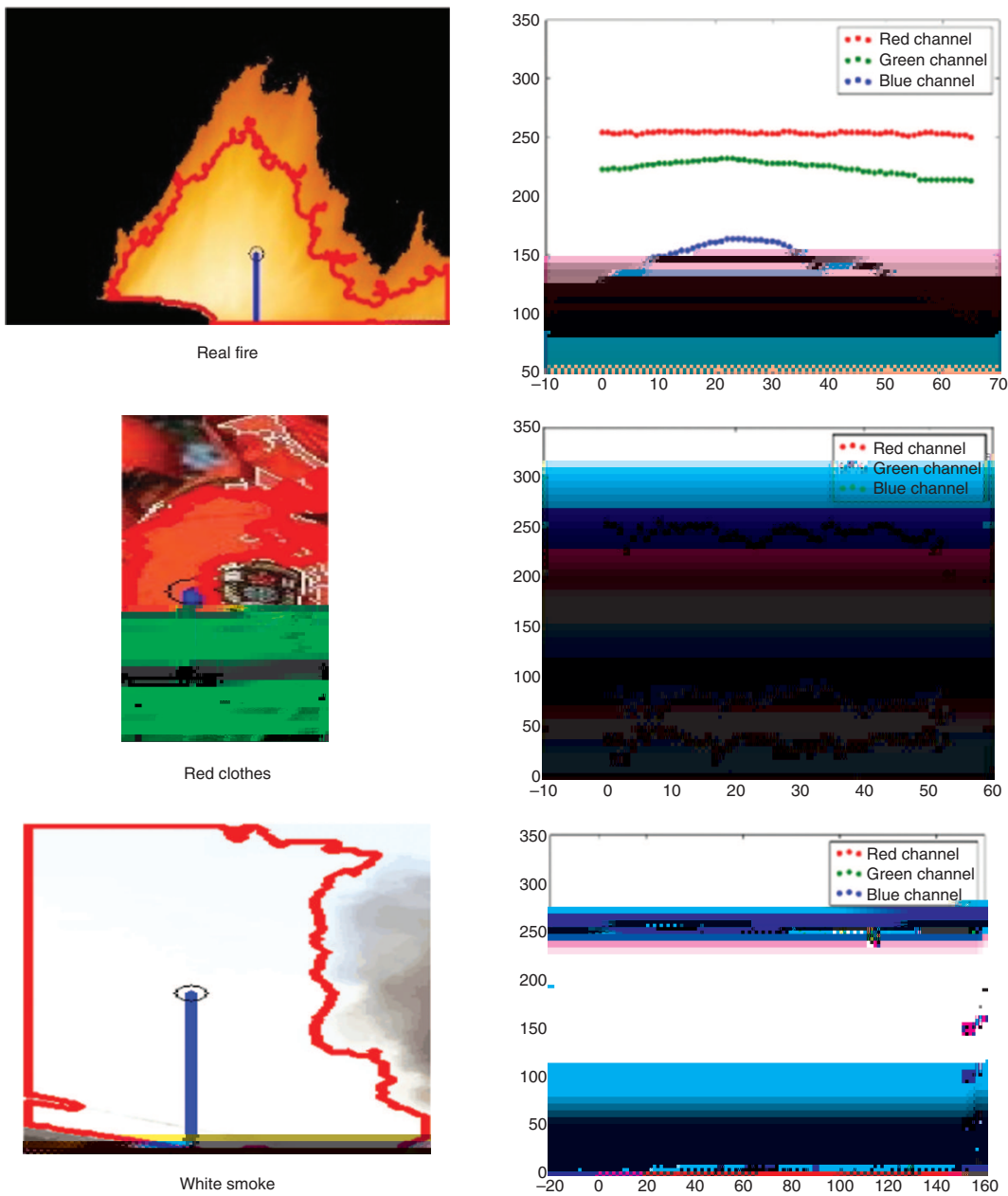

Figure 8: Changing Rate of the Pixel Value for Flame and Non-flame Regions.
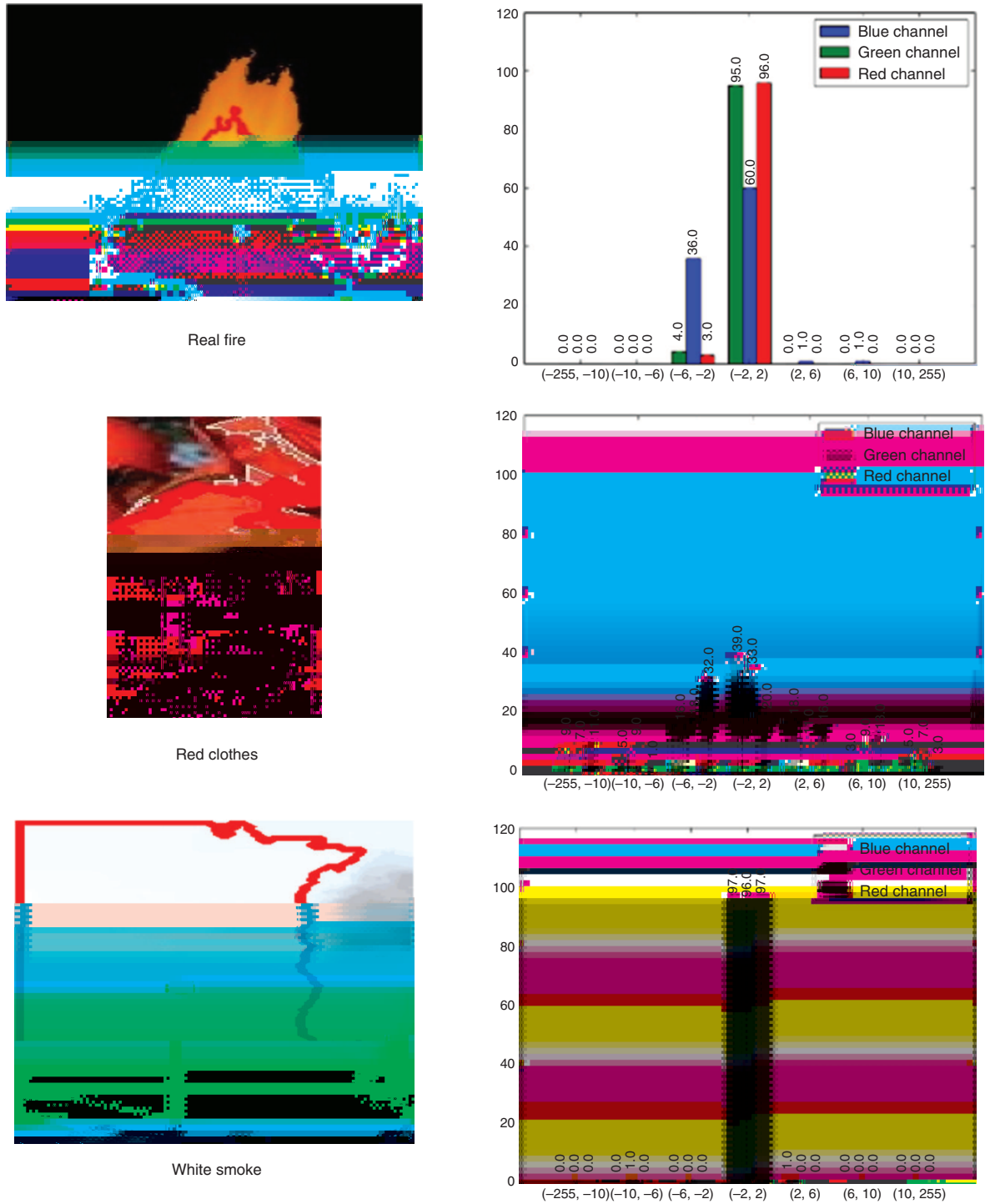
Real fire

Red clothes

White smoke

Figure 9: Pixel Value Gradient Distribution Range for Flame and Non-flame Regions.

the pixel value of the ame region has a wider range compared with the ame-like region, which could be represented as variations of the gradient in eight directions.

After calculating the distribution of color values in the RGB channels, the ratio of the mean probability density to the peak value was calculated based on probability density function [17].

In this study, we not only took into consideration the pixels of a flame but also calculated the pixel changes of a flame. As a result, the features that can reflect the changes of flame color and the association of RGB channels were designed based on the formulas defined as follows:

$$\overline{|\triangle R|} = \frac{1}{N}\sum_{n=1}^{N} |\triangle R_n|,$$ (1)

$$\sigma_{|\triangle R|} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(|\triangle R_n| - \overline{|\triangle R|}\right)},$$ (2)

$$\overline{|\triangle G|} = \frac{1}{N}\sum_{n=1}^{N} |\triangle G_n|,$$ (3)

$$\sigma_{|\triangle G|} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(|\triangle G_n| - \overline{|\triangle G|}\right)},$$ (4)

$$\overline{|\triangle B|} = \frac{1}{N}\sum_{n=1}^{N} |\triangle B_n|,$$ (5)

$$\sigma_{|\triangle B|} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(|\triangle B_n| - \overline{|\triangle B|}\right)},$$ (6)

$$\overline{\left|\frac{\triangle G}{\triangle R}\right|} = \frac{1}{N}\sum_{n=1}^{N}\left|\frac{\triangle G_n}{\triangle R_n}\right|,$$ (7)

$$\sigma_{|\frac{\triangle G}{\triangle R}|} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(\left|\frac{\triangle G_n}{\triangle R_n}\right| - \overline{\left|\frac{\triangle G}{\triangle R}\right|}\right)},$$ (8)

$$\overline{\left|\frac{\triangle B}{\triangle R}\right|} = \frac{1}{N}\sum_{n=1}^{N}\left|\frac{\triangle B_n}{\triangle R_n}\right|,$$ (9)

$$\sigma_{|\frac{\triangle B}{\triangle R}|} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(\left|\frac{\triangle B_n}{\triangle R_n}\right| - \overline{\left|\frac{\triangle B}{\triangle R}\right|}\right)},$$ (10)

where $\triangle R_n$ is the amount of change in the direction n in the R channel, $\overline{|\triangle R|}$ is the mean of $\triangle R_n$, $\sigma_{|\triangle R|}$ is the standard deviation of $\triangle R_n$, $\overline{|\triangle G|/|\triangle R|}$ is the mean of the ratio between the amount of change in channel G and channel R, and $\sigma_{|\triangle G|/|\triangle R|}$ is the standard deviation of $\overline{|\triangle G|/|\triangle R|}$. There are a total of 10 dimensions of the GF.

The decision tree algorithm was applied to the above-mentioned original features to classify the candidate region directly. However, poor experimental results were obtained based on the 10-dimension of these features. To achieve a better performance, we reduced the dimension of those features using PCA and generated six eigenvalues and eigenvectors [5]. Figure 10 shows the reduced feature for the flame region and non-flame region.

Table 2 summarizes the outcomes of our experiments. The extracted six eigenvalues accounted for 99.5% of the sum of all eigenvalues. Finally, we judged the color change sequence if it belongs to the flame, by using the decision tree model.

Figure 11 illustrates the final decision tree models trained in our experiments.

Figure 10: Values of GF.
Green represents features from flame regions; yellow represents features from non-flame regions.
(A), (B), and (C) are from different image samples.

Table 2: Eigenvalue and Eigenvector.

Eigenvalue														Eigenvector

Figure 11: Final Decision Tree in Our Experiments.

It is clear from Figure 11 that each node is the decision tree node. It means we split the decision space into two di erent regions by comparing it with the feature value. Take the rst node as an example, where the split feature is x_3 and the split value is -1.3096.

Experiments

. Data Description

The 500 sample ame images used in the experiment were taken from the Internet and consisted of pictures of forest res, bon re, candlelight, lights, tra c res, urban res, etc. These images can be divided into night-time ame images and daytime ame images according to their background scene. Therefore, this image set can validate the generalization of the algorithm.

. Choice of the Center Point for a Candidate Flame Region

In Figures 13–15, the red rectangle represents the candidate region as a ame region and the blue rectangle represents the candidate region as a non- ame region. In the experiment, it is di cult to nd the center of a ame, which is mostly identi ed by the eyes. To nd the center of the ame as soon as possible, we hypothe-sized that the observed ames are convex polygons. In this case, the centroid of the outline can be considered as the center of a ame, which can be calculated by using the cv2.moment module in OpenCV.

However, it is di cult to ensure that the centroid of the candidate re region contour is the center of the ame. As the centroid of the ame is used only for extracting the GF of the ame region, there is no need to nd the geometrical center of the ame precisely. As shown in Figure 12, the centroid of the region is not the geometrical center of the ame.

. Sample Analysis

It is di cult to identify the ame according to the change of the ame color exclusively. In the dark night, for example, when a ame or light is the main light source, nearby objects can be mapped to the corresponding color, such as the ground or people. However, it is relatively easy to identify the ame if the area is only a ame or a non- ame object. As shown in Figure 13, the outline of the picture is the area identi ed by SVM in the rst step.

Better results can be achieved if the ame area does not overlap with the non- ame area. However, in a real-world environment, as shown in Figure 14, the ame area and the non- ame area, whose color is similar to the ame, are intertwined.

Rigorously speaking, this kind of area is not a ame area because there is no ame in this area. However, in practical applications, a system should be timely alarmed if ame appears in the area, which is likely to be misinterpreted.

Moreover, if pixels of the image are low, lights at night are recognized (misinterpreted) as a ame, as shown in Figure 15. There is no better solution to this problem; it is yet to be improved.

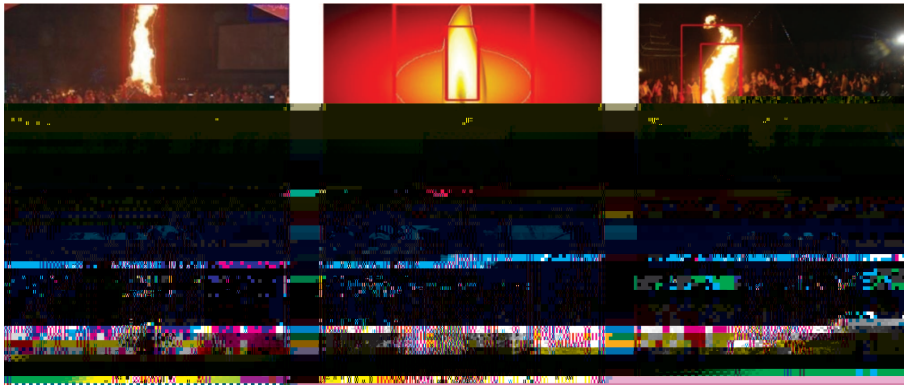**Figure 12:** Sample Images Whose Centroids are not in the Candidate Region.

Figure 13: Region Whose Color is Mapped to Flame Color.

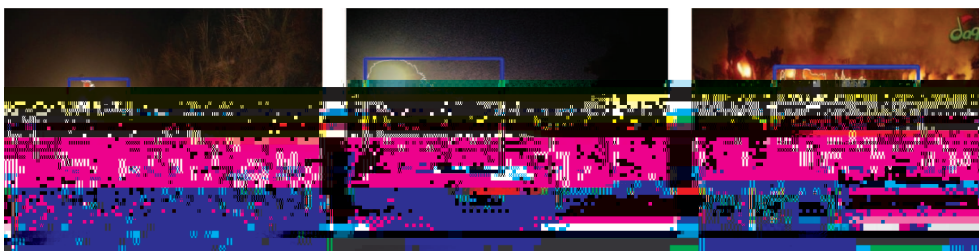**Figure 14:** Candidate Regions that Contain Flame or Non-flame.

Figure 15: Lights that are Misinterpreted as Flame.

.   Experimental Results

Some of the test samples did not contain ames but may contain objects that are easily misinterpreted as a ame, such as smoke and fog. Only ames were detected in this experiment, and smoke and fog were not

Table 3: Statistical Results.

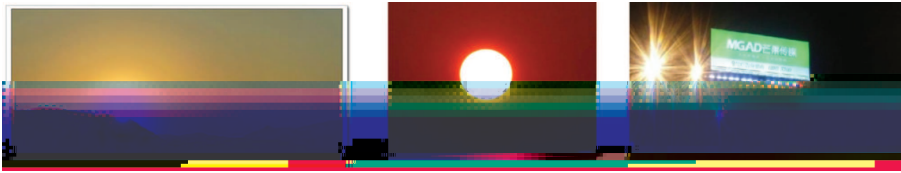| Proposed Method		Khatami et al. [ ]		SSD [ ]	
Precision	Recall	Precision	Recall	Precision	Recall
. %	. %	. %	. %	. %	. %

Figure 16: Some Successfully Discriminated Samples by Our Method.

detected. The sample images were crawled from the Web and may occasionally be repeated. As a comparison, the model with the highest accuracy in ames identi cation presented by Khatami et al. [8] in 2017 was used as the baseline. Meanwhile, as a popular deep learning method, a comparison with the regular target detection method, Single Shot Multibox Detector (SSD) [11], and the results obtained by Khatami et al. on the data set are shown in Table 3.

Table 3 presents the detailed data comparison based on a sample of 500 pictures. Two widely used evaluation metrics were applied in our experiments: precision and recall.

Compared to the method of Khatami et al. [8] and the SSD [11], the method presented in this paper had a better performance in the suspected ame area, such as smoke and other white objects such as candlelight mapping, the candle itself, red and yellow clothes, etc.

As is shown in Table 3, we achieved a better recall ratio and precision ratio compared to the other two methods due to the use of GF. Some successfully discriminated samples by our method are shown in Figure 16, which were recognized as a ame by the other two methods.

Conclusions

The proposed method consists of two main steps. The rst step is to nd the candidate ame region by applying a pre-trained classi er that aims to check whether a pixel is a ame or not. Each pixel of a picture will own a label of being ame or not. Based on the above results, a method of nding contour is applied to obtain the candidate region of ame. The second step is to check if the region is a ame or other things that look like a ame by extracting the GF.

The features of re ecting the pixel variations of the candidate ame region are extracted to di erentiate the ame region and regions like a ame. The principle behind this is the natural changes of the pixel of the   re region, which will show too strong or too mild changes for other ame-like objects such as smoke, fog, and red clothes. Experiments were conducted on some sample pictures. The experimental results demonstrated that the proposed method can e ectively distinguish the ame region from other ame-like regions.

In the rst step of our proposed method, it is necessary to check all the pixels of a picture; therefore, the speed is relatively slow. In the future, the way of checking each pixel in the rst step will be improved to accelerate the whole process.

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