Multi-View Non-negative Matrix Factorization Discriminant Learning via Cross Entropy Loss

Abstract:

Key Words:

1. INTRODUCTION

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2 - 0

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2. RELATED WORK

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10 , 1 - 21

, 22 - 2

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3. THE PROPOSED METHOD

3.1. Non-negative Matrix Factorization

$X \in \mathbb{R}_{+}^{m\times n}$,

n m

$W \in \mathbb{R}_{+}^{m\times k}$ $H \in \mathbb{R}_{+}^{n\times k}$

$$X \approx WH^{T} \quad (1)$$

$$\min_{W,H} \left\| X - WH^{T} \right\|_{F}^{2} \quad \text{s.t. } W,H \geq 0 \quad (2)$$

$\cdot_F$

W

H . , W H

$$\min_{W,H} \sum_{v=1}^{n_v} \left\| X^{v} - WH^{T} \right\|_{F}^{2} + \Phi(W,H) \quad \text{s.t. } W,H \geq 0$$

n_v , X^{v}

v- . Φ

W H .

3.2. Multi-view Learning via DICS

1 - ,

W_C

W_S^{v} v-

$X^{v} = W_C H_C^{T} + W_S^{v} (H_S^{v})^{T}$

$$W = W_{CD}\, W_{CN}\, W_{SD}^{v}\, W_{SN}^{v}$$

$$H = H_{CD}\, H_{CN}\, H_{SD}^{v}\, H_{SN}^{v}$$

W_{CD} H_{CD} , W_{CN} H_{CN} , W_{SD}^{v} H_{SD}^{v} , W_{SN}^{v} H_{SN}^{v} .

,

$B = B_{CD}\, B_{SD}^{v}$, H_{CD} H_{SD}^{v} .

$$\min_{W,H,B} \sum_{v=1}^{n_v} \left\| X^{v} - WH^{T} \right\|_F^2 + \Phi(W,H) + \gamma \left\| Y - \left[B_{CD}\; B_{SD}^{v} \right] \begin{bmatrix} H_{CD}^{T} \\ H_{SD}^{v\,T} \end{bmatrix} \right\|_F^2$$

$$\text{s.t. } W,H \geq 0$$

$Y \in \mathbb{R}^{c\times n}$, .

$y_{i,j} = 1$ j- i

$y_{i,j} = 0$.

3.3. Improved Objective Function via Cross-Entropy Loss

10

0 1,

0 1.

$$J = -\sum_{i=1}^{c} y_i \log p_i$$

c , y_i , y_i 1 0. p_i

$$\min_{W,H,B} \sum_{v=1}^{n_v} \left\| X^{v} - WH^{T} \right\|_F^2 + \alpha \left\| W_D^T W_D \right\|_{1,1} + \beta \left\| H_D \right\|_{1,1} - \gamma \sum_{j=1}^{n} \sum_{i=1}^{c} y_{ij} \log p_{ij} \quad (10)$$

α, β, γ - n_v , n , c .

$\|\cdot\|_{1,1}$ $l_{1,1}$

W_D,

$$\left\| W_D^T W_D \right\|_{1,1} = \sum_i w_{Di}^T w_{Di} + \sum_{i\neq j} w_{Di}^T w_{Dj} .$$

, $l_{1,1}$ H_D

p_{ij}

$$p_{ij} = \frac{e^{\sum_{k=1}^{k_1} b_{CD_{i,k}} h_{CD_{j,k}} + \sum_{k=1}^{k} b_{SD_{i,k}}^{v} h_{SD_{j,k}}^{v}}}{\sum_{t=1}^{n} e^{\sum_{k=1}^{k_1} b_{CD_{t,k}} h_{CD_{j,k}} + \sum_{k=1}^{k} b_{SD_{t,k}}^{v} h_{SD_{j,k}}^{v}}} \quad (11)$$

$f(W,H,B)$

$$\sum_{v=1}^{n_v} \left\| X^{v} - \sum_{i=1}^{k1} w_{CD_i} h_{CD_i}^T - \sum_{i=1}^{k2} w_{CN_i} h_{CN_i}^T - \sum_{i=1}^{k} w_{SD_i}^{v} h_{SD_i}^{v\,T} - \sum_{i=1}^{k} w_{SN_i}^{v} h_{SN_i}^{v\,T} \right\|_F^2$$

$$\alpha \left(\sum_{i=1}^{k1} \sum_{j=1}^{k1} w_{CD_i}^T w_{CD_j} + \sum_{i=1}^{k} \sum_{j=1}^{k} w_{SD_i}^{v\,T} w_{SD_j}^{v} + 2 \sum_{i=1}^{k1} \sum_{j=1}^{k} w_{CD_i}^T w_{SD_j}^{v} \right)$$

$$\beta 1_{1\times n} \sum_i h_{CD_i}$$

$1_{1\times n}$ n

1.

$$L = -\gamma \sum_{j=1}^{n} \sum_{i=1}^{c} y_{ij} \quad p_{ij}$$

$$p_{ij} = \frac{e^{z_{ij}}}{\sum_{t=1}^{n} e^{z_{tj}}} \quad 1$$

$$z_{ij} = \sum_{k=1}^{k_1} b_{CD_{i,k}} h_{CD_{j,k}} + \sum_{k=1}^{k} b^{v}_{SD_{i,k}} h^{v}_{CD_{j,k}}$$

$h_{CD_{i,j}}$

$$\frac{\partial L}{\partial h_{CD_{ij}}} = \frac{\partial L}{\partial p_{tj}} \cdot \frac{\partial p_{tj}}{\partial z_{ij}} \cdot \frac{\partial z_{ij}}{\partial h_{CD_{ij}}} \quad 1$$

$$\frac{\partial L}{\partial p_{tj}} = -\sum_{j=1}^{n} \sum_{i,t=1}^{c} \frac{y_{ij}}{p_{tj}} \quad 1$$

1 $t = i$

$$\frac{\partial p_{tj}}{\partial z_{ij}} = \frac{\partial \left(\frac{e^{z_{ij}}}{\sum_{t=1}^{c} e^{z_{tj}}} \right)}{\partial z_{ij}} = \frac{e^{z_{ij}} \cdot \sum_{t=1}^{c} e^{z_{tj}} - e^{z_{ij}} \cdot e^{z_{ij}}}{\left(\sum_{t=1}^{c} e^{z_{tj}}\right)^2} = p_{ij} \cdot (1 - p_{ij}) \quad 1$$

2 $t \neq i$

$$\frac{\partial p_{tj}}{\partial z_{ij}} = \frac{\partial \left(\frac{e^{z_{tj}}}{\sum_{t=1}^{c} e^{z_{tj}}} \right)}{\partial z_{ij}} = -\frac{e^{z_{tj}}}{(\sum_{t=1}^{c} e^{z_{tj}})^2} \cdot e^{z_{ij}} = -p_{tj} \cdot p_{ij} \quad 1$$

$b_{CD_{i,j}}$.

$$\begin{aligned}\frac{\partial L}{\partial h_{CD_{ij}}} &= -\sum_{j=1}^{n} \sum_{i,t=1}^{c} y_{ij} \cdot \frac{1}{p_{tj}} \cdot \frac{\partial p_{tj}}{\partial z_{ij}} \cdot b_{CD_{i,j}} \\ &= \sum_{j=1}^{n} \sum_{i,t=1}^{c} \left(-\frac{y_{ij}}{p_{ij}} \cdot p_{ij} \cdot (1 - p_{ij}) + \sum_{t \neq i} \frac{y_{tj}}{p_{tj}} \cdot p_{tj} \cdot p_{ij} \right) \cdot b_{CD_{i,j}} \\ &= \sum_{j=1}^{n} \sum_{i,t=1}^{c} \left(-y_{ij} + p_{ij} \sum_{t} y_{tj} \right) \cdot b_{CD_{i,j}}\end{aligned} \quad 1$$

Y 0 1 ,

$\sum_{t} y_{tj} = 1$.

$h_{CD_{i,j}}$

$$\begin{aligned}\frac{\partial L}{\partial h_{CD_{ij}}} &= \sum_{j=1}^{n} \sum_{i=1}^{c} \left(p_{ij} - y_{ij} \right) \cdot b_{CD_{i,j}} \\ &= \sum_{j=1}^{n} \sum_{i=1}^{c} \left(\frac{e^{\sum_{k=1}^{k_1} b_{CD_{i,k}} h_{CD_{j,k}} + \sum_{k=1}^{k} b^{v}_{SD_{i,k}} h^{v}_{SD_{j,k}}}}{\sum_{t=1}^{c} e^{\sum_{k=1}^{k_1} b_{CD_{t,k}} h_{CD_{j,k}} + \sum_{k=1}^{k} b^{v}_{SD_{t,k}} h^{v}_{SD_{j,k}}}} - y_{ij} \right) \cdot b_{CD_{i,j}}\end{aligned} \quad 1$$

h_{CD_i}

$\frac{\partial L}{\partial h_{CD_i}} = \sum_{i=1}^{n} \left(p_i - y_i \right)^{T} \cdot b_{CD_i}$, p_i

$+$ $max(0,\cdot)$,

η

$$w_{CD_i} = w_{CD_i} + \eta \left[\sum_{v=1}^{n_v} \left(R^{v} h_{CD_i} - \alpha \left(W_{CD} 1_{k1\times1} + W^{v}_{SD} 1_{k \times1} \right) \right) \right]_{+} \quad 20$$

$$w_{CN_i} = w_{CN_i} + \eta \left[\sum_{v=1}^{n_v} R^{v} h_{CN_i} \right]_{+} \quad 21$$

$$w^{v}_{SD_i} = w^{v}_{SD_i} + \eta \left[R^{v} h^{v}_{SD_i} - \alpha \left(W_{CD} 1_{k1\times1} + W^{v}_{SD} 1_{k \times1} \right) \right]_{+} \quad 22$$

$$w^{v}_{SN_i} = w^{v}_{SN_i} + \eta \left[R^{v} h^{v}_{SN_i} \right]_{+} \quad 2$$

$$h_{CD_i} = h_{CD_i} + \eta \left[\sum_{v=1}^{n_v} \left(R^{v} w_{CD_i} - \frac{\beta}{2} 1_{n\times1} - \frac{\gamma}{2} Q^{vT} b_{CD_i} \right) \right]_{+} \quad 2$$

$$h_{CN_i} = h_{CN_i} + \eta \left[\sum_{v}^{n_v} R^{vT} w_{CN_i} \right]_{+} \quad 2$$

$$h^{v}_{SD_i} = h^{v}_{SD_i} + \eta \left[R^{vT} w^{v}_{SD_i} - \frac{\beta}{2} 1_{n\times1} - \frac{\gamma}{2} Q^{vT} b^{v}_{SD} \right]_{+} \quad 2$$

$$h^{v}_{SN_i} = h^{v}_{SN_i} + \eta \left[R^{vT} w^{v}_{SN_i} \right]_{+} \quad 2$$

R^{v} Q^{v}

R^{v} X^{v} $-W_{CD}H_{CD}^{T} - W_{CD}H_{CD}^{T}$ $-W_{SD}^{v}H_{SD}^{v\,T} - W_{SN}^{v}H_{SN}^{v\,T}$ 2

Q^{v} = $B_{CD}H_{CD}^{T} + B_{SD}^{v}H_{SD}^{v\,T}$ $-Y$ 2

η ,

, B_{CD} B_{SD}^{v}

B_{CD} B_{SD}^{v}

$$B_{CD} = \frac{1}{n_v}\sum_{v=1}^{n_v}\left(Y - B_{SD}^{v}H_{SD}^{v\,T}\right)H_{CD}\left(H_{CD}^{T}H_{CD} + \lambda I\right)^{-1} \quad 0$$

$$B_{SD}^{v} = \left(Y - B_{CD}H_{CD}^{T}\right)H_{SD}^{v}\left(H_{SD}^{v\,T}H_{SD}^{v} + \lambda I\right)^{-1} \quad 1$$

I λ

4. EXPERIMENT

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4.1. Datasets

1.

1.

	1200			2000
	2	2		201
	1	2		1 0
	1	2		1 0 1
	2 0	2		1 0 0
	2	2		1 0

$k = 1$

λ 0. 0.

λ $10^0, 10^1, 10^2, 10$, 10

11 ,

λ $10^0, 10^{-1}, 10^{-2}, 10^{-}$

2 ,

α 100, β γ 0,100,200, 00,1000

α 100 0 2 0 , β 0.01, 0.02, 0.0 , γ 0.00

α β $[0,1]$, γ 1.

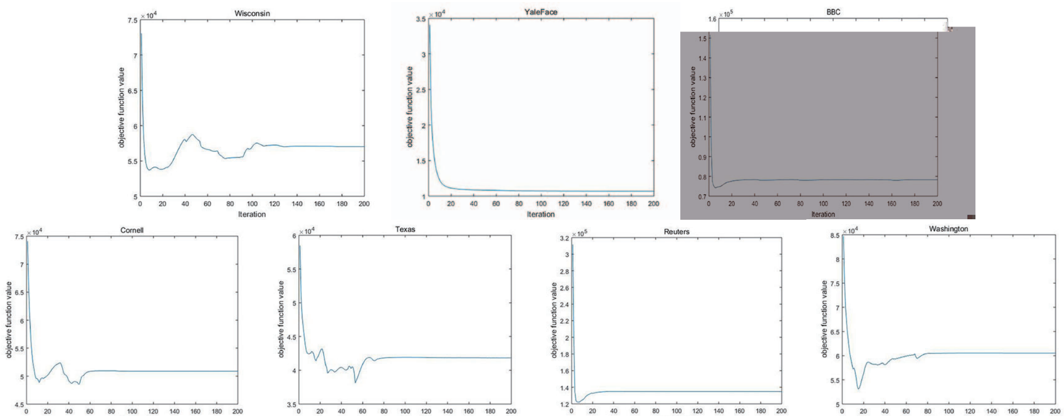
4.2. Result

10

2

2.

	0. 1.2	0.0 2.	.0 1.	1.0 1.	. 1.	. 2.2	2. 1.
	2. 0.2	.2 .2	.1 0.2	. .	. .	. 2.	0. .
	. 1.	. .	. 2.	0. .0	. .	2. .	. 2.
	0. .0	.1 .2	0.2 2.	2. .1	1. .0	. .0	.1 .
	0. 1.	0.2 1.	1. 1.2	. 2.2	. 2.1	. 2.2	. 1.



1.

4.3. Convergence Analysis

2,

1,

100

0

4.4. Discriminant Matrix Visualization

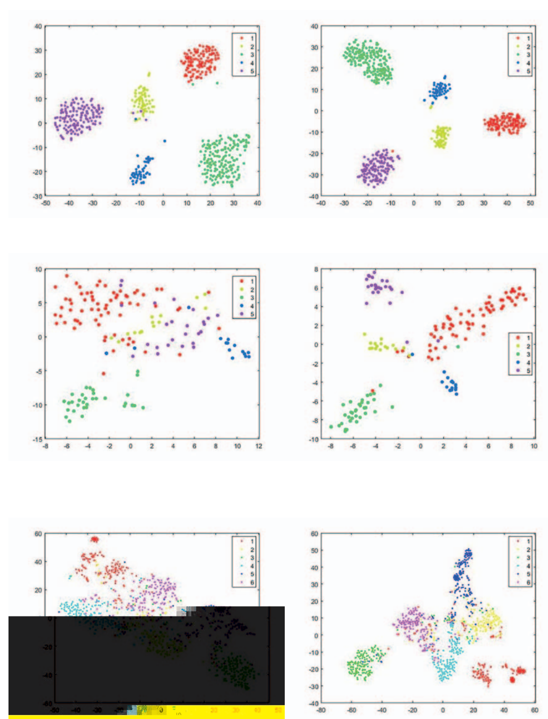
200

2

W_D

H_D

X



2.

5. Conclusion and Future Work

REFERENCES

1 ... *Proceedings of the eleventh annual conference on Computational learning theory*, ... 1 ... 2-100.

2 ... *Proceedings of the 24th international conference on Machine learning*, ... 200 ... 11 -11 .

1 . , , , , ,
Breakthroughs in statistics, , , , 1 2,
. 1 2-1 0.

1 . . . , . . , . , . - , ,
. , -2 ,
, *Advances in Neural Information Processing Systems*, , , , 200 , .
2.

1 . . , . . - ,
, *Neural Computation*, . 1 , . 12, .
2 2 , . 200 .

20 , . , . , . . , -
, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, . , . 1, .
1 1 , . 201 .

21 . , . , . . . ,
, *2012 IEEE Conference on Computer Vision and Pattern Recognition*, , , 2012, . 21 0-21 .

22 . . , , *ICML workshop on learning with multiple views*, 200 , . 20 2 .

2 . , . . , - -
- , *Proceedings of the Twenty-Fifth International Joint Conference on Artificial Intelligence*, , , , 201 , .
1 1-1 .

2 . , . , . . , ,
, *IEEE Transactions on Systems, Man, and Cybernetics*, , . 0, . , .
1 -1 , . 2010.

2 . . . . , ,
, *Machine Learning, Proceedings of the Twenty-Fourth International Conference*, , , , 200 , . 11 11 .

2 . , . . , -
,
IEEE 16th International Conference on Data Mining (ICDM), , , 201 , . 12 12 0.

2 . . Alpaydın,
, *Journal of machine learning research*, . 12,
. 2211-22 , . 2011.

2 . . . , -
- , *Proceedings of the 28th International Conference on Machine Learning*, . ,
, , 2011, . 00.

2 . . . , - ,
Proceedings of the 27th International Conference on Machine Learning, , , 2010, . 11 11 2.

0 . . , -
, *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, . 1, . , .
1 12-1 2 , . 2011.

1 . , . , . , . . ,
, *Proceedings of the 16th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, , , , 2010, .
11 11 .

2 . , . , . . ,
, *Data Mining and Knowledge Discovery*, . 2 , . 1, . ,
. 201 .

. , . , . , . . . ,
,
Proceedings of the 21th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining,
, , . , 201 , . .

. , ., , ., , ., . , . ., . , .
. . 2 ,
12 12 201

. , . . , - ,
, *IEEE Signal Processing Letters*,
. 1 , . 1, . - , . 2010.

. . , . - ,
Journal of machine learning research, . , . 2 -2 0 ,
. 200 .