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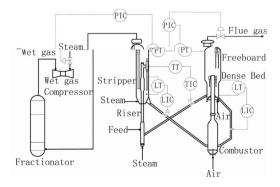
ABSTRACT

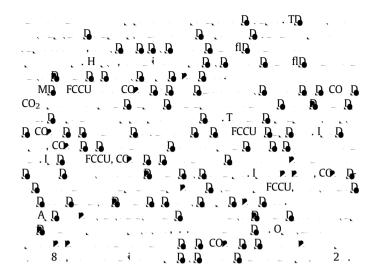
1. Introduction

D CO. D CO. (NLP). A 23.24. **Q** 27,28. NLP, 29,30 **NSGBD** NCO-T. , 🖟 🖟 _ FCCU fi,

2. Batch properties of FCCU

7,34-36 . A . 1. H 37,





3. Hybrid parametric dynamic optimization

3.1. Mathematical formulation of hybrid parametric dynamic optimization

$$\dot{\boldsymbol{x}}_d = \boldsymbol{f}_d \Big(t, \boldsymbol{x}_d(t), \boldsymbol{x}_d(t), \boldsymbol{u}(t), \overline{\boldsymbol{u}} \Big) \tag{1}_-$$

$$\mathbf{0} = \mathbf{f}_a \Big(t, \mathbf{x}_d(t), \mathbf{x}_a(t), \mathbf{u}(t), \overline{\mathbf{u}} \Big) \tag{1}$$

$$\int J(\boldsymbol{u}(t),\overline{\boldsymbol{u}}) = \int\limits_{t_0}^{t_f} (-r(t,x_d(t),x_a(t),\boldsymbol{u},\overline{\boldsymbol{u}}) + c_1(t,x_d(t),x_a(t),\boldsymbol{u},\overline{\boldsymbol{u}}))dt + c_2(\overline{\boldsymbol{u}})$$

$$\tilde{\mathbf{x}} = -\mathbf{r} + \mathbf{c}_1 \tag{3}$$

 $\tilde{x}(t_0) = 0$ (4

$$X(t_0) = 0 \tag{4}$$

 \mathbb{P} \mathbb{P}

$$u(t), \overline{u}J\left(x(t_f), \overline{u}) = \tilde{x}(t_f) + c_2(\overline{u})$$
 (5)

 $\mathbf{x}(t) = (\mathbf{x}_d(t)^T, \mathbf{x}_d(t)^T)^T.$

$$x^{lb} \le x(t) \le x^{ub} \tag{6}_{-}$$

$$u^{lb} \le u(t) \le u^{ub}$$
 (6)

$$\overline{u}^{lb} \leq \overline{u} \leq \overline{u}^{ub} \tag{7}$$

 \overline{u}^{b} \overline{u}^{ab} \overline{u}^{ab} \overline{u}^{b} \overline{u}^{b

$$\mathbf{x}^{flb} \leq \mathbf{x}(t_f) \leq \mathbf{x}^{fub} \tag{8}$$

 $x^{flb} = ((x_d^{flb})^T, (x_a^{flb})^T)^T, x^{fub} = ((x_d^{fub})^T, (x_a^{fub})^T)^T, x_d^{flb} - x_d^{fub} -$

Problem (P1):

$$u(t), \overline{u} \int (x(t_f), \overline{u})$$
 (9_)

$$\dots x_d(t_0) = x_d^0 \tag{9}$$

$$\dot{\boldsymbol{x}}_{d} = \boldsymbol{f}_{d} \left(\boldsymbol{t}, \boldsymbol{x}_{d}(t), \boldsymbol{x}_{a}(t), \boldsymbol{u}(t), \overline{\boldsymbol{u}} \right) \tag{9.}$$

$$\mathbf{0} = f_a \Big(t, \mathbf{x}_d(t), \mathbf{x}_a(t), \mathbf{u}(t), \overline{\mathbf{u}} \Big) \tag{9}$$

$$G_{\overline{u}}(\overline{u}) \leq 0$$
 (9)

$$G_{u}(u(t)) \leq 0 \tag{9}$$

$$G_p(x(t)) \le 0 \tag{9}$$

$$G_{e}\left(x(t_{f})\right) \leq 0 \tag{9}$$

$$G_{\overline{u}}(\overline{u}) = \begin{pmatrix} \overline{u} - \overline{u}^{ub} \\ \overline{u}^{lb} - \overline{u} \end{pmatrix} \tag{9}$$

$$G_{u}\left(u(t)\right) = \begin{pmatrix} u(t) - u^{ub} \\ u^{lb} - u(t) \end{pmatrix} \tag{9}$$

$$G_p(x(t)) = \begin{pmatrix} x(t) - x^{ub} \\ x^{lb} - x(t) \end{pmatrix}$$

$$\tag{9}$$

$$G_{e}\left(x(t_{f})\right) = \begin{pmatrix} x(t_{f}) - x^{fub} \\ x^{flb} - x(t_{f}) \end{pmatrix}$$

$$\tag{9}$$

(11)

 \mathbf{D} \mathbf{D} $\mathbf{u}(t) \approx \widehat{\mathbf{u}}^N(t) = \sum\nolimits_{i=1}^N \mathbf{u}_i^N \boldsymbol{\phi}_i^N(t)$

 $\forall i \neq j$. H , , , $<\phi_i^N(t), \phi_i^N(t)> = 0$ D $\forall i \neq j$ D D_N . Fig.

 \mathbf{D}_{-} , \mathbf{p}_{i} $\phi_{i}^{N}(t)$, . $\phi_i^N(t) = \begin{cases} 1, t_{i-1} \leq t \leq t_i \\ 0, \end{cases}$

 $\widehat{u}_N = ((u_1^N)^T, ..., (u_N^N)^T)^T$ $\widehat{u}_N = (NLP. S, P. 1..., D. 1$ $y^l(t) = y^u(t)$ $y^{lb}-y(t)$ $\dot{y}^{u} = (0,y(t)-y^{ub}),$ $y^{l}(t_{0}) = 0$, $y^{u}(t_{0}) = 0$, $y^{u}(t_{f}) = 0$. MD D ,

Problem (P2):

$$\sum_{u_N,\overline{u}} J(x(t_f),\overline{u}) \tag{12}$$

.
$$MX(t_0) = X_0$$
 (12)

$$M\dot{X}(t) = F(t, X(t), \hat{u}_N, \overline{u})$$
 (12,)

$$G(\overline{u}, \widehat{u}_N, X(t_f)) \le 0 \tag{12}$$

 $M = \underbrace{(\underbrace{1,....,1}_{3n_d+2n_a},\underbrace{0,...,0}_{n_a})}$ (12)

$$X(t) = \begin{pmatrix} x_d(t) \\ x^b(t) \\ x_a(t) \end{pmatrix}$$
 (12)

$$X_0 = \begin{pmatrix} x_d^0 \\ 0 \\ 0 \end{pmatrix} \tag{12}$$

$$F(t, X(t), \widehat{u}_N, \overline{u}) = \begin{pmatrix} f_d(t, x_d(t), x_a(t), \widehat{u}_N, \overline{u}^j) \\ f_b(t, x(t)) \\ f_a(t, x_d(t), x_a(t), \widehat{u}_N, \overline{u}^j) \end{pmatrix}$$
(12)

$$f_b(t, \mathbf{x}(t)) = \begin{pmatrix} & & (\mathbf{0}, \mathbf{x}^{lb} - \mathbf{x}(t)) \\ & & (\mathbf{0}, \mathbf{x}(t) - \mathbf{x}^{ub}) \end{pmatrix}$$
(12)

 $G(\overline{u}, \widehat{u}_N, X(t_f)) = \begin{pmatrix} G_u(u) \\ G'_u(\widehat{u}_N) \\ G'_e(x^b(t_f)) \\ G_e(x(t_f)) \end{pmatrix}$ (12)

$$G'_{u}(\widehat{\boldsymbol{u}}_{N}) = \begin{pmatrix} G_{u}(\boldsymbol{u}_{1}^{N}) \\ \vdots \\ G_{u}(\boldsymbol{u}_{N}^{N}) \end{pmatrix}$$
 (12)

$$G'_e\left(x^b(t_f)\right) = x^b(t_f) \tag{12}$$

3.2. Nonconvex sensitivity-based generalized Benders decomposition

 $h(\overline{u},\underline{u}) = \underline{u} - \overline{u} = 0$ (13)

(P2),_, ,**D**

$$(\widehat{\boldsymbol{u}}_{N,a}^{j}|J_{a}^{j}) = \lim_{n \to \infty} \int_{\widehat{\boldsymbol{u}}_{N}} J\left(\boldsymbol{x}(t_{f}), \overline{\boldsymbol{u}}_{a}^{j}\right)$$

$$(14_{-})$$

$$\dots MX(t_0) = X_0 \tag{14}$$

$$M\dot{X}(t) = F\left(t, X(t), \hat{u}_N, \bar{u}^i\right)$$
 (14,)

$$G\left(\overline{u}_{a}^{j}, \widehat{u}_{N}, X(t_{f})\right) \leq 0 \tag{14}$$

$$(\widehat{\boldsymbol{u}}_{N,b}^{j}, \overline{\boldsymbol{u}}_{b}^{j} | \boldsymbol{\mu}_{a}^{j}) = \underline{\qquad} \widehat{\boldsymbol{\omega}}_{N,\overline{\boldsymbol{u}}} J \left(\boldsymbol{x}(t_{f}), \overline{\boldsymbol{u}} \right)$$

$$(15_{-})$$

$$\dots MX(t_0) = X_0 \tag{15}$$

$$M\dot{X}(t) = F\left(t, X(t), \widehat{u}_N, \overline{u}\right)$$
 (15,)

$$G(\overline{u}, \widehat{u}_N, X(t_f)) \leq 0 \tag{15}$$

$$h\left(\overline{u},\underline{u}_{a}^{j}\right)=0 \tag{15}$$

$$\mu_{a}^{j} \tag{15}.$$

(16)

(16)

 (17_{-})

(17)

Problem (P5):

$$(\widehat{\boldsymbol{u}}_{N,c}^{j'}, \overline{\boldsymbol{u}}_{c}^{j'}) = \sum_{\boldsymbol{u} \in \overline{\boldsymbol{u}}} \| \overline{\boldsymbol{u}} - \overline{\boldsymbol{u}}_{t}^{j} \|_{A}^{2}$$

$$(\widehat{\boldsymbol{u}}_{N,c}^{j}, \overline{\boldsymbol{u}}_{c}^{j}) = \underline{\qquad} \widehat{\boldsymbol{u}}_{N,\overline{\boldsymbol{u}}} \| \overline{\boldsymbol{u}} - \overline{\boldsymbol{u}}_{t}^{j} \|_{A}^{2}$$

$$(16_{-})$$

$$\dots MX(t_0) = X_0$$

$$M\dot{X}(t) = F(t, X(t), \widehat{u}_N, \overline{u})$$

$$G(\overline{u}, \widehat{u}_N, X(t_f)) \leq 0$$

 $\|\overline{\boldsymbol{u}} - \overline{\boldsymbol{u}}^j\|_A^2 = (\overline{\boldsymbol{u}} - \overline{\boldsymbol{u}}^j)^T A (\overline{\boldsymbol{u}} - \overline{\boldsymbol{u}}^j)$

A ____ fi **Problem (P6):**

$$\dots MX(t_0) = X_0$$

$$M\dot{X}(t) = F(t, X(t), \hat{u}_N, \overline{u})$$
 (17,)

$$G(\overline{u}, \widehat{u}_N, X(t_f)) - \alpha e \le 0 \tag{17}$$

$$h(\overline{u}, \overline{u}_C^{\dagger}) = 0 \tag{17}$$

$$\boldsymbol{\mu}_{c}^{i} \cdot \left(\underline{\boldsymbol{u}} - \overline{\boldsymbol{u}}^{ub}\right) \leq 0, i \in K_{u} \tag{18}_{-}$$

$$\boldsymbol{\mu}_{d}^{i'} \cdot \left(\underline{\boldsymbol{u}} - \overline{\boldsymbol{u}}^{lb} \right) \leq 0, i' \in K_{l}$$
 (18)

$$\mu_c^i = (\underbrace{0,...,0}_{i-1},1,\underbrace{0...,0}_{Li})_{-,} \quad \mu_d^{i'} = (\underbrace{0,...,0}_{i-1},-1,\underbrace{0...,0}_{Li'}).$$
 Problem (P7):

$$\left(\eta_b^k, \underline{u}_b^k | \mathbf{v}_a^k, \mathbf{v}_b^k, \mathbf{v}_c^k, \mathbf{v}_d^k\right) = \prod_{n, \mathbf{v}} \eta \tag{19}_{-}$$

$$\eta \ge J_a^j + \mu_a^j \cdot \left(\underline{u} - \overline{u}_a^j\right), j \in K_{feas}$$
(19)

$$\boldsymbol{\mu}_{b}^{j'} \cdot \left(\underline{\boldsymbol{u}} - \overline{\boldsymbol{u}}_{c}^{j'} \right) \leq 0, j' \in K_{infeas} \tag{19},$$

$$\boldsymbol{\mu}_{c}^{i} \cdot \left(\underline{\boldsymbol{u}} - \overline{\boldsymbol{u}}^{ub}\right) \leq 0, i \in K_{u} \tag{19}$$

$$\boldsymbol{\mu}_{d}^{i'} \cdot \left(\underline{\boldsymbol{u}} - \overline{\boldsymbol{u}}^{lb} \right) \leq 0, i' \in K_{l}$$
 (19)

 $\mathbf{v}_a^k, \mathbf{v}_b^k, \mathbf{v}_c^k = \mathbf{v}_d^k = \mathbf{L}_{a}$ $K_{u} = \mathbf{K}_b$ K_{b}

$$\Xi_a = \{ j \in : (\boldsymbol{v}_a^k)_i \neq 0 \} \tag{20}_-$$

$$\Xi_b = \{j' \in : (v_b^k)_{j'} \neq 0\}$$
 (20)

$$\Xi_c = \{i \in : (\boldsymbol{v}_c^k)_i \neq 0\}$$
 (20,)

$$\Xi_d = \{i' \in : (v_d^k)_{i'} \neq 0\}$$
 (20)

Problem (P8):

$$(\underline{\boldsymbol{u}}_{c}^{k}) = \mu \boldsymbol{\mu}_{\underline{\boldsymbol{u}}}^{k} \boldsymbol{\mu}_{\underline{\boldsymbol{u}}}^{k} \cdot \underline{\boldsymbol{u}}$$
 (21_)

(16)
$$\boldsymbol{\mu}_b^{j'} \cdot (\underline{\boldsymbol{u}} - \overline{\boldsymbol{u}}_c^{j'}) \leq 0, j' \in K_{infeas}$$
 (21)

$$(16,) \qquad \boldsymbol{\mu}_{c}^{i} \cdot (\underline{\boldsymbol{u}} - \overline{\boldsymbol{u}}_{u}) \leq 0, i \in K_{u}$$
 (21,)

$$\boldsymbol{\mu}_{d}^{i'} \cdot (\boldsymbol{u} - \overline{\boldsymbol{u}}_{l}) \leq 0, i' \in K_{l} \tag{21}$$

$$T = \{x, y, z\}$$

$$LBD > UBD \tag{22}_{_})$$

$$UBD - LBD < \varepsilon_1 \tag{22}$$

$$\|\boldsymbol{\mu}_{a}^{k}/J_{a}^{k}\|_{B} \leq \varepsilon_{2} \tag{23}$$

$$\|\underline{\boldsymbol{u}}_{h}^{k} - \underline{\boldsymbol{u}}_{c}^{k}\|_{B} \leq \varepsilon_{3} \tag{23}$$

Algorithm 1: NSGBD with CVP for Problem (P1)

 $b t_0, t_f, t_0 t_1 < t_1 < t_2 < \ldots < t_N = t_f, t_0$

 \mathbf{D} \mathbf{D} \mathbf{D} \mathbf{D} \mathbf{D} \mathbf{D} \mathbf{U} \mathbf{U} E. . (7); $j = 1, j' = 1, K_{feas} = \emptyset, K_{infeas} = \emptyset, LBD = -\infty;$

Step 2. SQ. PQ. (P3)
$$\overline{u}_a^j = \overline{u}_t^j$$
. O. Q. Q. Q. Q. (P3)

(1). P
$$\longrightarrow$$
 Q (P3) \longrightarrow $\widehat{u}_{N,a}^j \longrightarrow \widehat{J}_a^j$; Q \longrightarrow \longrightarrow P Q (P4) Q $\underline{u}_a^j = \overline{u}_a^j \longrightarrow \overline{u}_a^j$, $\widehat{u}_{N,a}^j \longrightarrow \mu_a^j$. UBD $= J_a^j$; Q Q

$$\begin{array}{ccc}
\mathbf{D} & \mathbf{D} & \vdots \\
\mathbf{O} & \mathsf{LBD} > \mathsf{UBD}. & \boldsymbol{\mu}_a^l = \gamma \boldsymbol{\mu}_a^l, l \in \Xi_a.
\end{array}$$

(2). UBD – LBD
$$< \varepsilon_1$$
. O, \square , \square

$$(P8) = \overline{u}_a' \qquad \underline{u}_c' \cdot 1 \quad E. \quad (23) \quad \dots \quad \text{if} \quad (\underline{u}_b' = \overline{u}_a'), \quad \dots \quad \dots \quad D.$$

$$(3) \cdot 0 \quad \dots \quad \dots \quad D.$$

$$(3) \cdot 0 \quad \dots \quad \dots \quad D.$$

$$K_{feas} = \{K_{feas}, j\}; j = j + 1.$$

Step 3. Sp. P D (P7)
$$\eta_b^i, \underline{u}_b^i, v_a^j, v_b^i, v_b^i = v_d^i$$

$$LBD = \eta_b^I, \overline{u}_t^I = \underline{u}_b^I. R \qquad \square S \triangleright 2.$$

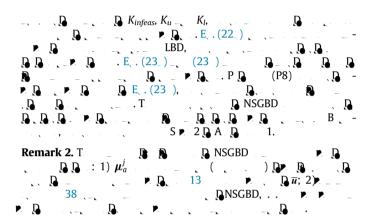
P. . _ P 🖟 .

D D D (P3). P D (P5)_ (P6)_

 \mathbf{P} \mathbf{Q} $\mathbf{u}_c^{j'}$

(P6) D α = 0. P Ω (P7)

 γ \square LBD Ξ_b,Ξ_c Ξ_{d-}



3.3. Novel implementation framework of optimal solution



4. Hybrid parametric dynamic optimization of FCCU

4.1. Mathematical formulation of FCCU

(26)

4.2. Case 1: Combustion air as a continuous operation whereas CO promoter as a batch operation

C P NLP. D 32 NSGBD Ω ε_1 =1, ε_2 = 0.1 _ NSGBD_ $M_{pro}^5 = 3.1864$ ⁵ = 1655.2¥. T . . **.** . . NCO-D D fi. $L = k \in \mathbb{N}$: $\mu^{k} < 0$, $R = k \in \mathbb{N}$: $\mu^{k} > 0$, D $M^k_{pro} < M^*_{pro}$, $k \in L_-$, $M^k_{pro} > M^*_{pro}$, $k \in R. H_-$, , D D $\sum_{k \in R} M_{pro}^k$ (3.1864, 3.3440) $M_{pro}^* \in (\bigcup_{k \in L} M_{pro}^k,$ (27)**3.3**, D NCO-D . T Α $3, ... M_{pro} = 3.1864$ $M_{pro} = 3.3440$ NCO-**D** . B 3.344, 3.45 . T $M_{pro} = 3.4kg$ I = 2732.3¥



1 _ 16	4.4	3	O	/	0	9	10				
$M_{pro}($	2.5865	2.4590	2.7855	2.8738	2.8890	3.2639	3.3223				
μ_{M}	8.9744	-21.541	27.047	25.754	20.927	53.115	58.887				
$V_1(-3/1)$	49.034	49.061	49.064	49.064	49.065	49.068	49.068				
μ_{V1}	-3480.3	-3515.4	-3471.9	-3479.8	-3490.8	-3480.0	-3468.3				
$V_2(-3/1)$	48.952	48.977	48.978	48.979	48.979	48.980	48.981				
# √2	-3456.5	-349EL57295	50TD X4747F 111	Γ. 773316705Γ D(3447. 3 / 161.9 1T	4.973041208T.ID()Te	FB11B4 372 B41	D TD(3476.1)	F.5151T 5.54	840TD()T /F11	T .764
	3841 0 T8 9										

$$M_{\text{pro}}^* \in (10^{\circ} \text{ Mpro}, M_{\text{pro}}^{10}) = (2.5865, 3.3223)$$

$$(28_{-})$$

$$V_1^* \in (V_1^{10}, \quad _{-k \in N_1} V_1^k) = (49.068, 49.072) \quad ^3/$$
 (28)

$$V_2^* \in (V_2^{10}, _{-k \in N_2} V_2^k) = (48.981, 48.983)$$
 ³/

$$V_3^* \in (V_3^{10}, \dots, V_3^k) = (48.931, 48.932) \dots^3 /$$
 (28)

(28)

 $V_4^* \in (V_4^{10}, \dots, V_4^k) = (48.896, 48.897) \times {}^3/.$

