Shortest Uplink Scheduling for NOMA-Based Industrial Wireless Networks

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Abstract—The power-domain nonorthogonal multiple access (NOMA) based successive interference cancellation (SIC) ... the problem of the shortest uplink scheduling (SUS) ... by both the power allocation and WS scheduling. A new term named maximum decoded level (MDL), which models the transmitting characteristics of WSs under SIC ... For the case of access point adapted 2-SIC ... power allocation ... k-SIC ...

Index Terms—Nonorthogonal multiple access (NOMA), successive interference cancellation, power, scheduling, and access.

I. Introduction

IN RECENT years, wireless networks are playing more and more important roles in industries. Distinct from the cellular networks such as long term evolution-advanced, where downlinks carry more traffics than uplinks, in industrial wireless networks (IWNs), a sink usually collects sensory data from wireless sensors (WSs), thus, the performances of uplinks are vital for IWNs [1], [2].

Since the real-time performance with guaranteed delay is often required in IWNs [3], the problem of the shortest uplink scheduling (SUS), i.e., how to minimize the length of the uplink frame with given traffic loads, has to be tackled. Relative to the random media access, the classic time division multiple access (TDMA) technique has the advantage of bounded access time, which is the product of the slot span and the frame length. However, the uplink frame length could be terribly large especially for the high-density heavy-load IWNs, which will be common with further development of Internet of Things. Therefore, better solutions have to be sought.

Power-domain nonorthogonal multiple access (NOMA) is one of the candidate solutions to the next-generation IWNs. Successive interference cancellation (SIC), which is the one important implementation method of the power-domain NOMA.[1] Nowadays, supports parallel transmissions from multiple transmitters by multiplexing in the power domains [4] and, thus, has great potentials for low-delay applications. Therefore, the problem of finding the SUS strategy if SIC-based sink is adapted in IWNs, has to be solved.

We solve the problem by joint power allocation and WS scheduling. On one hand, WS scheduling determines how to group the WSs, since WSs in a group will transmit simultaneously, and thus, they will interfere with each other. On the other hand, the power allocation sets reasonable transmit powers for WSs, so that the transmitted symbols from the WSs in a group can be decoded by an SIC-based sink without errors.

We first investigate the SUS problem,[2] when transmit powers of WSs are continuously adjustable. First, a key term named maximum decoded level (MDL), which models the transmitting characteristics of WSs under SIC, is defined. Based on MDL, an important characteristic, i.e., the so-called power exclusiveness, is revealed, which lays theoretical foundations for a sufficient and necessary condition for successful transmissions under SIC. The sufficient and necessary condition directly results in the decoupling between WS scheduling and power allocation, which is the key outcome of the first step. Based on the outcome, we present a two-step greedy algorithm for the SUS problem. We also prove that the algorithm is optimal for two most regular cases as follows. One is for any traffic loads under 2-SIC, the other is for unit traffic load under k-SIC. Besides, an explicit analytic expression of the optimal solution is also presented for the both cases, respectively.

In view of the above-mentioned results obtained, we further investigate the same problem, however, with discrete transmit

[1]An inherited flaw of power-domain NOMA is its high-power consumption, and therefore, it is suitable for some industrial applications, which requires low access delays and no stringent power constraints [3]. Besides, extra decoding delay due to the successive decoding process is also inevitable.

[2]Although the scheduling-based policies are beneficial for enhancing the network performance in general cases, it cannot provide real-time processing for burst events, such as the retransmissions.

powers.[3] We also propose an optimal algorithm under 2-SIC, and a heuristic algorithm under k-SIC, respectively, based on the greedy algorithm proposed in the section of continuous transmit powers.

Our major contributions are summarized as follows.

1) As to the SUS problem for SIC-based IWNs with given traffic loads, we formulate it by joint power allocation and WS scheduling.

2) We define MDL and then reveal a sufficient and necessary condition for successful parallel transmissions under SIC, which decouples the WS scheduling from the power allocation. What is more, since the so-called power exclusiveness perfectly depicts the decoding feature of SIC, we believe that it can also be utilized in other SIC-related problems.

3) For general cases, a greedy algorithm is proposed. However, it is also optimal for two regular cases.

4) For the case of discrete transmit powers, we also propose an optimal algorithm under 2-SIC and a heuristic algorithm under k-SIC.

The remainder of this article is organized as follows. Section II reviews the related works, and Section III introduces the system models. Problem formulation and solutions are introduced and analyzed in Section IV. Based on the conclusions drawn in Section IV, the same problem with discrete transmit powers is considered in Section V. Performance evaluations are in Section VI, and Section VII concludes this article.

II. RELATED WORKS

NOMA schemes, which are categorized into power-domain NOMA and code-domain NOMA, can be used in the scenarios of single-antenna and multiple-input–multiple-output [5], [6]. Scheduling for performances enhancement is a classic topic in network research works. Nowadays, Scheduling for performances enhancement based on NOMA has attracted great attention in both industry and academia. For example, the power-domain NOMA, which is based on SIC receivers, is now under full consideration for industrial applications or heterogeneous cellular networks [7]. The classic maximum weight schedule has been proven to be the maximum throughput schedule under the primary interference model [8]. As for the minimum length schedule problem, related works can be differentiated from three aspects. The first is the underlying interference models including the protocol interference model and the physical interference model. The second is the network scenario including the single-hop and the ad hoc networks, and the third is the transmit rate and power models adopted, including the signal-interference-plus-noise ratio (SINR) based and the fixed-value-based. For example, the work in[9] is under the physical interference model, for the ad-hoc networks, using the SINR-based rate model and the continuously adjustable transmit power model. Similar

SIC based Sink

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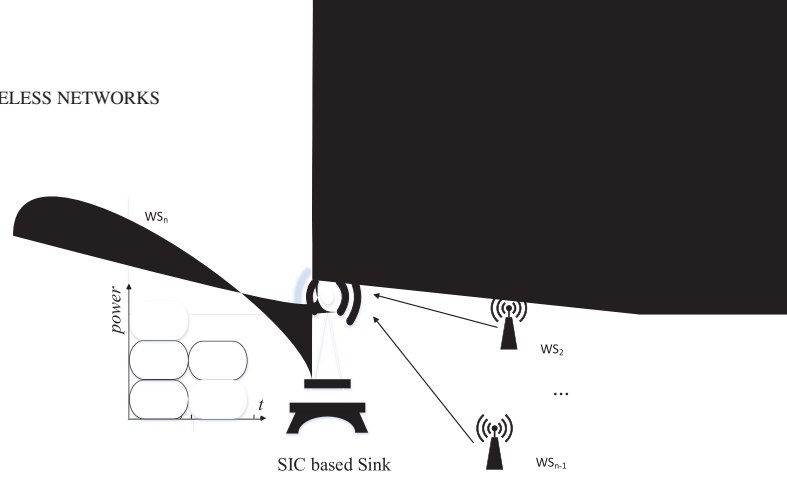
[3]Generally, the transmit powers are not continuously adjustable. Take TI CC1000 transceiver for example. The transmit power is programmable, and there are 30 programmable settings for output levels in the step of 1 dBm. We have a research on the cases of continuous transmit powers because it lays theoretic foundations for the case of discrete transmit powers.

TABLE I
Notations

| WS_t | |

words, to find the optimal strategy, we only need to find the WS scheduling strategy, which achieves the shortest length without taking power allocation into considerations.

Intuitively, for a WS that has smaller channel gain and smaller maximal transmit power ceiling, its scheduling flexibility is obviously weaker in SIC decoding schemes. To find the optimal solution to SUS-kSIC, the scheduling flexibility of WS under SIC has to be modeled mathematically. Evidently, under SIC, the transmit power ceiling, the channel gain, and the decoding threshold jointly affect the scheduling flexibility of WSs. The term MDL is set up to model the scheduling flexibility of WSs under SIC.

fi 2 Power Threshold Sequence for r-SIC (PTS-r) is a sequence $\widehat{X} = (\widehat{X}_1, \widehat{X}_2, \ldots, \widehat{X})$, which satisfies the following equality group:

$$\begin{cases} \dfrac{\widehat{X}}{\sum_{=1}^{1} \widehat{X} + n_0} = \gamma & j \quad [2, r] \\ \dfrac{\widehat{X}_1}{n_0} = \gamma \end{cases}$$

where $\widehat{X} > 0$ for all $j \geq 1$ and $\gamma > 1$.

Obviously, PTS-r is a geometric sequence. An explicit formula for PTS-r is as follows: $\widehat{X}_1 = \gamma n_0, \widehat{X}_{+1} = (\gamma + 1)\widehat{X}$ for $i \quad [1, r \quad 1]$. PTS-r is in fact the minimum received powers required for r signals if the r signals are to be successfully decoded by a k-SIC receiver where $k \geq r$.

1 For the following inequality group:

$$\begin{cases} \dfrac{x}{\sum_{=1}^{1} x + n_0} \geq \gamma & l \quad [2, r] \\ \dfrac{x_1}{n_0} \geq \gamma \end{cases} \tag{2}$$

any of its solution $(\widetilde{X}_1, \widetilde{X}_2, \ldots, \widetilde{X})$ satisfies $\widetilde{X} \geq \widehat{X}$ for $i \quad [1, r]$, where $\widehat{X} = (\widehat{X}_1, \widehat{X}_2, \ldots, \widehat{X})$ is PTS-r.

It is easy to prove using mathematical induction. Please refer to Appendix A. ∎

fi 3 (ᵀ ᵀ) For WS, whose channel gain to the sink is G and its transmit power ceiling is $p^{\max}$, if there exists an integer l, such that $\widehat{X} \leq p^{\max}G \leq \widehat{X}_{+1}$, the MDL of WS is l.

Intuitively, MDL models the interference tolerance capability of WSs. For example, for the WS with MDL=1, it can only be decoded in the first decoding phase, because it has very weak immunity from interferences. Obviously, the larger is its MDL of a WS, the more decoding phases it can choose to be decoded in.

ᵀ 1 (ᵀ ᵀ k-) Given PTS-k being $(\widehat{X}_1, \widehat{X}_2, \ldots, \widehat{X})$, provided that the following two prerequisites are satisfied, the number of parallel WSs, i.e., WSs which transmits simultaneously, is at most l.
1) Transmissions from different WSs can be decoded simultaneously by the k-SIC based sink.
2) The maximum of all received powers lies in $[\widehat{X}, \widehat{X}_{+1}]$, where $l \leq k$.

Please refer to Appendix B. ∎

Lemma 1 is for revealing the following two characteristics of the optimal solution from the perspective of MDL.

ᵀ 2 (ᵀ ᵀ ᵀ) Assume that w packets from w WSs transmit simultaneously where $w \leq k$. All of these w packets can be decoded correctly by a k-SIC-based sink, if and only if the MDL of the WS, which is decoded in decoding phase i, must be no less than i for any $i \quad [1, w]$.

Please refer to Appendix C. ∎

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Based on the above-mentioned conclusion, we only need to focus on the WS scheduling strategy that achieves the minimum scheduling length. Algorithm 1 is a greedy algorithm based on Lemma 2.

The WS scheduling strategy is generated slot by slot. In determining the scheduling strategy for every slot, there are three stages including the anchoring, the upper phase allocation and the lower phase allocation. In the anchoring stage, the WS with the heaviest loads is selected and assigned to the anchoring phase equal to its MDL. In the upper phase allocation stage, some WSs are selected and assigned to the upper phases. Similar process is done for the lower phases in the lower phase allocation stage.

Lines 1 and 3 are for initialization. Lines 4–8 is the so-called anchoring process, where we compare the traffic load of the unscheduled type-1 WS ensemble with that of every other WS, and choose one WS, i.e., $usr1$ in the algorithm, based on the criterion that the WS with the heaviest traffic load is preferred.[11] In line 7, a decoding phase, i.e., phs_init in the algorithm, is reserved for the chosen WS, i.e., $usr1$, based on its MDL value. The so-called upper phases assignment is from lines 9–11, where we choose the WS, i.e., $usr2$ in the algorithm, for the decoding phase larger than phs_idx. The process goes on until no suitable WS is found. Similar process, i.e., the lower phases assignment, is from lines 12–15, where we choose the WS, i.e., $usr3$ in the algorithm, for the decoding phases less than phs_idx.[12]

After Algorithm 1, we set powers for WSs based on their phases allocated. For u, if its phase allocated is j, its transmit power is set as $\widehat{X}/G$. The correctness of the power allocation strategy is guaranteed by Lemma 2.

The following example under 4-SIC is presented for an overview of the algorithm. There are five WSs, u_1 to u_5, whose MDLs and traffic loads are shown in the left upper corner of Fig. 2. For the first slot, u_2 is selected as $usr1$ and reserved the decoding phase 2 since its MDL is 2. Next, we choose WSs for the phases larger than 2, i.e., phases 3 and 4. For phase 3, u_5 is chosen based on the criterion depicted by line 10. For phase 4, we cannot find an eligible WS because MDLs of the remaining WSs are all less than 4. Furthermore, we choose a WS for phases less than 2, i.e., phase 1. u_3 is, thus, chosen based on

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[11]The reason for selecting the WS that has the heaviest unscheduled load is to balance the unscheduled traffic loads among the type-1 WSs ensemble and every other WS. In that way, smaller frame length will be resulted in.

[12]The reason for arranging decoding phases larger than phs_init prior to these less than phs_init is to stuff as many WSs in a slot as possible, because WSs with larger MDL have larger scheduling flexibility.
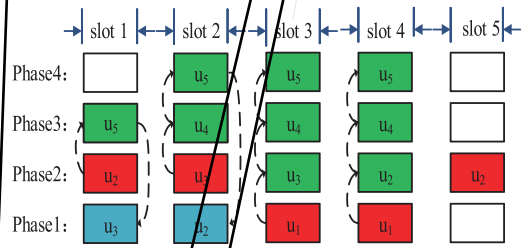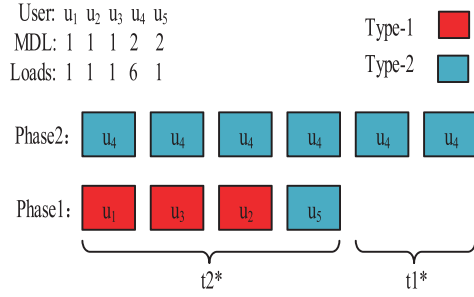
Fig. 4. Example illustrates Proposition 2.

T_1 is the number of noncompound slots and T_2 is the number of compound slots.

Please refer to Appendix D. ∎

4 For the WS scheduling strategy output by Algorithm 1, if there are more than one noncompound slots, they must be monopolized either by the type-1 WSs, or by a same WS of type-2.

Please refer to Appendix E. ∎

To prove the optimality of Algorithm 1 in the special case 1, two extra propositions are presented. Note that the two propositions further deepen the conclusion of Lemma 4.

1 For the WS scheduling strategy output by Algorithm 1, if there are more than one noncompound slot, and every noncompound slot is monopolized by a type-1 WS, then for any compound slot, it always contains a type-1 WS.[15]

Please refer to Appendix F. ∎

2 For the WS scheduling strategy output by Algorithm 1, if there are more than one noncompound slot, and all noncompound slots are monopolized by a same type-2 WS, w.l.o.g., assume the type-2 WS is u, then u will be contained in every compound slot.[16]

Please refer to Appendix G. ∎

2 Algorithm 1 outputs an optimal solution to SUS-2SIC.

We prove it in two cases using notations in Lemma 3.

1 $T_1 = 1$, i.e., there is only one noncompound slot in the WS scheduling strategy output by Algorithm 1. In this case, the total traffic load, i.e., $(\sum_{i=1} L)$, must be odd, therefore, the minimum frame length in theory is $\frac{(\sum_{i=1}^n i)}{2}$, where is for upper rounding. On the other hand, since $T_1 = 1$, the frame length output by Algorithm 1 is also $\frac{(\sum_{i=1}^n i)}{2}$. Thus, the WS scheduling strategy output by Algorithm 1 is the optimal in this case.

2 $T_1 > 1$. Based on the conclusion of Lemma 3, the case could be further put into two subcases as follows.

Subcase 2.1: Every noncompound slot is monopolized by a type-1 WS. In this subcase, based on Proposition 1, the frame length by Algorithm 1 is equal to the load sum of the type-1 WS ensemble. Therefore, the WS scheduling strategy output by Algorithm 1 is the optimal in this subcase.

[15]The proposition can be verified by the example in Fig. 3.
[16]The proposition can be verified by the example in Fig. 4.

Subcase 2.2: All noncompound slots are monopolized by the same type-2 WS, and w.l.o.g., denote the WS by u. In this subcase, the minimum frame length is obviously no less than the traffic load of u. On the other hand, the length of the WS scheduling strategy output by Algorithm 1 is equal to the traffic load of u. Therefore, the WS scheduling strategy output by Algorithm 1 is also optimal in this subcase. ∎

In the special case, the time complexity of Algorithm 1 is $O(\sum_{=1} L)$. It has linear complexity with traffic loads.

2) ⊢ ⊢ *2* ⊢ *fi* ⊢ *k-* The special case where every WS can only transmit once and only once in a frame often takes place in IWNs. We prove that Algorithm 1 is also the optimal in this case. Besides, a closed-form expression of the shortest frame length is also presented.

⊢ *5* In the special case where $L = 1$ for all i $[1, n]$, for the WS scheduling strategy output by Algorithm 1, if the last slot contains a WS, which is scheduled at the jth decoding phase, then in all other slots, there are always j WSs, which will be scheduled from the first to the jth phase, respectively, and besides, all of their MDLs are no larger than j.

Please refer to Appendix H. ∎

3 For the special case of SUS-kSIC, where $L = 1$ for all i $[1, n]$, Algorithm 1 outputs an optimal WS scheduling strategy.

Please refer to Appendix I. ∎

4 For the special case of SUS-kSIC, where $L = 1$ for all i $[1, n]$, the shortest frame length is max $\left\{ \frac{1}{1}, \frac{1 + 2}{2}, \ldots, \frac{\sum_{i=1}^k i}{} \right\}$, where n denotes the number of type-i WSs.

Please refer to Appendix J. ∎

Theorem 4 can be understood in a more intuitive manner as follows. For the type-1 WSs, they can only be decoded in the decoding phase 1, thus, there are at least n_1 slots in a frame. while for the WSs whose MDLs are no greater than w, they can be assigned to any decoding phase from 1 to w, thus, at least $\sum_{i=1}^w i$ slots have to be contained in a frame, and so forth. Therefore, the shortest scheduling length $T_{\min}$ is no less than max $\left\{ \frac{1}{1}, \frac{1 + 2}{2}, \ldots, \frac{\sum_{i=1}^k i}{} \right\}$.

V. SUS WITH DISCRETE POWER FOR k-SIC

Assume there are m transmit power levels $\overline{tp}, \overline{tp}_1, \ldots, \overline{tp}_1$ where $\overline{tp} > \overline{tp}_1 > \cdots > \overline{tp}_1$, and $\frac{i+1}{i} = q$ for i $[1, m \quad 1]$. They consist of a feasible power set $\overline{TP} = \overline{tp}, \overline{tp}_1, \ldots, \overline{tp}_1$. W.l.o.g., assume $p^{\max}$ $\overline{TP}$ for all i $[1, n]$. The SUS with discrete power for k-SIC (SUSDP-kSIC) problem is formulated as follows:

$$\min_{ij \quad ij} t \tag{3a}$$

$$\text{s.t.} \quad (1b); \quad (1c); \quad (1\,d); \quad (1e); \quad (1f) \tag{3b}$$

$$p \quad \overline{TP} \text{ for } i \quad [1, n] \quad j \quad [1, t]. \tag{3c}$$

Obviously, relative to (1), the continuous power cases, only the extra constraint (3c), which is the constraint of feasible transmit power, is appended.
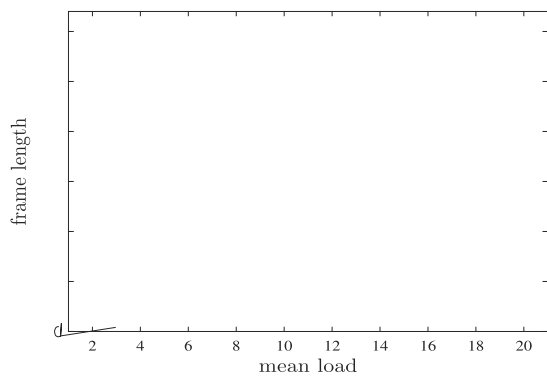
TABLE IV
FRAME LENGTH UNDER 2-SIC RECEIVER

edge length / μ	125	250	500	1000
2	30.148 (0.150)	30.148 (0.150)	30.148 (0.150)	30.15 (0.154)
8	120.383 (5.308)	120.383 (5.308)	120.383 (5.308)	120.394 (5.487)
12	180.239 (11.436)	180.239 (11.436)	180.239 (11.436)	180.256 (11.516)
16	240.228 (20.476)	240.228 (20.476)	240.228 (20.476)	240.263 (20.884)
20	300.229 (28.047)	300.229 (28.046)	300.229 (28.046)	300.285 (28.400)

TABLE V
SCHEDULING CAPABILITY FOR CON

k-SIC / μ	2-SIC	3-S
2	Z	
4	Z	
6	S	
8	I	
10	I	
12	I	

TABLE
PERFORMANCE OF THRO

k-SIC / μ	TDMA	2-
2	29.471	14.

better scheduling flexibility, and therefore, shorter frame length, just as revealed in Table III.

For the two cases where the edge length is 125 m and 250 m, respectively, their frame lengths are the same. Similar results can also be found in other cases of traffic loads. The phenomenon reveals that the distribution of MDLs instead of the value of MDLs plays a deterministic role in the frame length. In fact, for the above-mentioned four cases of edge lengths, the MDL distributions of WSs, i.e., the proportion of WSs whose MDLs are equal to 1, 2, 3, 4 are (0, 0, 0, 1), (0, 0, 0, 1), (0, 0.023, 0.438, 0.539), and (0.264, 0.422, 0.180, 0.134), respectively. The results also defend that the same MDL distribution results in the same frame length.

To have further verifications, we repeat all experiments under 2-SIC and list results in Table IV. We also take the case $\mu = 2$ in Table IV as an example, the frame lengths when edge length is 125 m, 250 m, and 500 m are all the same, since the MDL distributions in the three cases are the same for 2-SIC receiver. Their distinctions from these in 4-SIC receiver reveal that for the same topology, the MDL distribution of WSs will be more balanced for k-SIC receivers when k is smaller. The similar phenomena can also be found for other traffic load cases. The reason is easy to be understood since for a k-SIC receiver, there are k levels of MDLs. An extreme example where $k = 1$ will be helpful for understanding. The MDLs of all WSs are all 1 in this case, i.e., their MDLs are completely balanced. In one word, the larger the k, the more unbalanced the MDL distributions.

In all the above-mentioned experiments, for analyzing factors influencing the frame length performance, the traffic load for a WS is allocated in one time, i.e., the traffic loads are given before determining the scheduling strategy. However, in practical scenarios, data are always generated continuously. To have an objective evaluation for practical scenarios, we assume that the buffer of any WS is infinite, and data are generated continuously and stored in buffers until they are scheduled for transmitting.
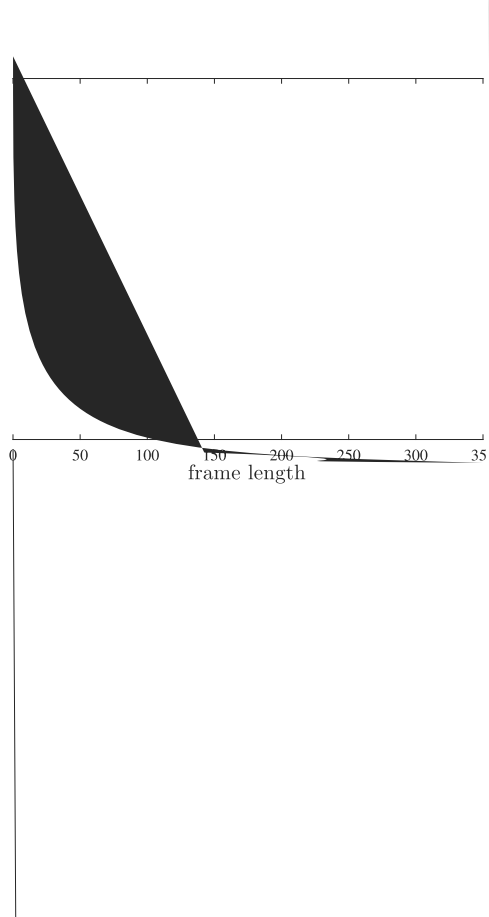
Based on [8], the network is thought to be balanced if the volume of data buffered in every buffer is finite after infinite

frame length

parallel WSs that are decoded in phase $[1, i - 1]$. Therefore, the two inferences contradict.

fi For the WS, which is successfully decoded in decoding phase i, say WS , since its MDL is no less than i, i.e., $\widehat{}_j^i \le P^{\max}$, we can set its transmit power as $\widehat{}_j^i$. We do the similar power allocation for every $i \in [1, w]$. It can be verified that SINR of every WS is γ, that is, all of these w packets can be decoded correctly. ∎

. 3

The lemma can be proven by contradictions. Assume it is not the fact, there are two slots, w.l.o.g., assume they are the jth and the $(j+1)$th slot, where the jth slot is noncompound while the $(j+1)$th is compound.

1 The MDL of the WS in the jth slot is 1. The emergence of the $(j+1)$th slot that is compound is impossible by Algorithm 1, or else the type-2 WS in the $(j+1)$th slot must be scheduled in the jth slot according to Algorithm 1.

2 The MDL of the WS in the jth slot is 2. The emergence of the $(j+1)$th slot is impossible, because either the type-1 WS or the type-2 WS must be scheduled in the jth slot according to Algorithm 1. ∎

. 4

The lemma can be proven by contradictions. For two noncompound slots, if they are monopolized by a type-1 and a type-2 WS, respectively, they will be combined as one compound slot according to Algorithm 1. In other words, the multiple noncompound slots could not be monopolized simultaneously by type-1 and type-2 WSs. Furthermore, if two noncompound slots are monopolized by two distinct type-2 WSs, the two WSs must be same, or else they will be combined as one compound slot by Algorithm 1. ∎

. 1

Based on the same notations and the conclusion of Lemma 4, the first T_2 slots are compound and the remaining T_1 slots are noncompound. Therefore, at the beginning of the T_2th slot, the unscheduled traffic load of type-1 WS ensemble is larger than that of any a type-2 WS, because there are more than one noncompound slots, which are monopolized by type-1 WSs. Based on the lines 4–6 of Algorithm 1, a type-1 WS will be chosen as $usr1$, i.e., the T_2th slot must contain a type-1 WS. Thus, at the beginning of the $(T_2 - 1)$th slot, the traffic load of all type-1 WS ensemble is larger than that of any type-2 WS. The above-mentioned procedure goes iteratively until the beginning of the first slot. The proposition is thus proven. ∎

. 2

Based on the same notations and the conclusion of Lemma 4, the first T_2 slots are compound and the remaining T_1 slots are noncompound. And, all noncompound slots are monopolized by u . Therefore, at the beginning of the T_2th slot, the unscheduled traffic load of u is not only larger than that of any other type-2 WS but also larger than the unscheduled traffic loads of type-1 WS ensemble. Based on the lines 4–6 of Algorithm 1, u will be chosen as $usr1$, i.e., the T_2th slot must contain u . Thus, at the beginning of the $(T_2 - 1)$th slot, the unscheduled traffic load of u is not only larger than that of any other type-2 WS but also larger than the unscheduled traffic loads of type-1 WS ensemble. The above-mentioned procedure goes iteratively until the beginning of the first slot. The proposition is thus proven. ∎

. 5

It is easy to be proven by contradictions. For brief, notate the WS by u . First, there is no empty phase from 1 to j in any slot except for the last slot, or else, u will be allocated to the empty phase according to Algorithm 1.

Second, the MDL of these WSs allocated to phases 1 to j in every slot except for the last one are no larger than j. Or else, w.l.o.g., assume u , whose MDL is greater than j, is among them. In this case, u would not be chosen when Algorithm 1 chooses a WS for the position of u , since u have higher priority than u based on lines 6, 10, or 14 in Algorithm 1. Therefore, u could not be contained in the last slot. ∎

. 3

We prove it by contradictions. Assume the optimal slot number is T_{opt}. Therefore, the frame length by Algorithm 1 is at least $T_{\text{opt}} + 1$. W.l.o.g., if the MDL of the variable $usr1$ in the $T_{\text{opt}} + 1$th slot is j, based on Lemma 5, there are at least $jT_{\text{opt}} + 1$ WSs whose MDL is no larger than j. So, according to lemma 2, the frame length of any a feasible sensor scheduling strategy, including the optimal one, is at least $\frac{(_{\text{opt}}+1)}{}$, i.e., $T_{\text{opt}} + 1$, which contradicts the assumption. ∎

. 4

According to Lemma 5, if the MDL of the $usr1$ in the last slot is j, the scheduling length is $\sum_{i=1}^{j} {}_i$. We now try to prove that $\sum_{i=1}^{j} {}_i = \max \left\{ \frac{{}_1}{1}, \frac{{}_1 + {}_2}{2}, \ldots, \frac{\sum_{i=1}^{k} {}_i}{} \right\}$

Notate $\sum_{i=1}^{j} {}_i$ by $T_{\min}$.

1 For all $1 \le l \le j$

Based on Lemma 5, $\sum_{=1} n \le lT_{\min}$, i.e., $T_{\min} \ge \frac{\sum_{i=1}^{l} {}_i}{}$. Therefore, $T_{\min} \ge \frac{\sum_{i=1}^{l} {}_i}{}$ since $T_{\min}$ is an integer.

2 for all $j + 1 \le l \le k$

Case 2.1. MDLs of all WSs allocated for phase $j \sim k$ in the last slot are distinct.

Based on Lemma 5, $\sum_{=1} n = l(T_{\min} - 1) + 2$. Therefore, $T_{\min} \ge \frac{\sum_{i=1}^{l} {}_i}{}$ since $l \ge 2$.

Case 2.2. MDLs of all WSs allocated for phase $j \sim k$ in the last slot are not distinct.

Based on Lemma 5, $\sum_{=1} n \le l T_{\min}$, and $\sum_{=1} n \quad 2 \ge$ $(T_{\min} \quad 1)l$. Therefore, $\frac{\sum_{i=1}^{l} i}{} \le T_{\min} \le \frac{(\sum_{i=1}^{l} i)\ 2+}{}$. So, $T_{\min} = \frac{\sum_{i=1}^{l} i}{}$, since $l \ge 2$.

In conclusions, the frame length of the optimal sensor scheduling strategy is $\max\left\{\frac{1}{1}\ ,\ \frac{1+\ 2}{2}\ ,\ldots,\ \frac{\sum_{i=1}^{k} i}{}\right\}$ ∎

. ⊢ 6

$$\frac{}{\sum_{j=1}^{j=i} \frac{i}{j+}\ ^1} \ge \frac{}{\frac{a_i}{q+1} + \frac{a_i}{(q+1)^2} + \ \frac{i\ a_i}{(q+1)^{i}\ ^1} + \frac{a_i}{(q+1)^{i}\ ^1 q}}$$

$$\ge \frac{}{\frac{\sum_{j=1}^{j=i}\frac{a_i}{(q+1)^j}}{}} \ge \frac{}{\frac{\sum_{j=}^{i}\frac{a_i}{(q+1)^j}}{}} \ge q. \ \blacksquare$$

. 5

⊢ 1 There is only one WS u in the slot. Its transmit power is $\min\left\{p^{\max}, tp_1\right\}$, and the transmission from u can be decoded correctly.

⊢ 2 There is more than one WS in the slot. W.l.o.g., assume two WSs, u and u, are allocated to phase $l+1$ and l by line 9 of Algorithm 3, respectively. In this case, $\frac{tp_{l+1}*\ i}{tp*\ j} = \frac{[\![tp_l * \frac{G_{\max}}{G_{\min}} * (1+\ )]\!] *\ i}{tp_l *\ j} \ge$ $\frac{tp_l * \frac{G_{\max}}{G_{\min}} * (1+\ ) *\ i}{tp_l *\ j} = \frac{\max* i}{\min* j} * (1+\gamma) \ge (1+\gamma)$.

Besides, $\frac{\min\ ^{\max}_{i}\ 1}{0} \ge \gamma$ always holds. Therefore, based on Lemma 6, all parallel transmissions are successful. ∎

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