



# Wiener structure based adaptive control for dynamic processes with approximate monotonic nonlinearities

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## Abstract

The modeling of industrial processes requires to consider the complex features of systems, such as nonlinearities, dynamics and uncertainties, etc. In this paper, a simplified Wiener structure (SWS) is proposed for the modeling of dynamic processes with approximate monotonic nonlinearities. The nominal SWS not only considers the dynamic characteristics in processes, but also takes full advantages of the process nonlinear properties. Then, an adaptive control method for the SWS is proposed, in order to achieve exact output tracking of reference signals in the servo control mode. The recursive estimation is implemented before the adaptive control law is ready. In the recursive computation, the ideas of both the discrete Nussbaum gain and the dead-zone factor are introduced. The tracking theorem verifies the stability of adaptive control under the suitable assumption. Finally, two illustrative examples demonstrate the effectiveness of both the SWS and the adaptive control.

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## 1. Introduction

Since most industrial processes have both nonlinear and dynamic properties, it is necessary to build full dynamic models. Although there are many methods for modeling nonlinear dynamic processes [1–6], they meet the same problem: the mutual fusion relationship between dynamics and nonlinearities. This leads to inadequate and inaccurate modeling of processes [7,8]. Further, the separated modeling of dynamics and nonlinearities is recommended.

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That is, the nonlinear block-oriented models are adopted for process modeling. There are three types of block-oriented structures: Hammerstein structure [9], Wiener structure [10], and their combinations [11]. Hammerstein structure, i.e., the input-nonlinear form, can describe systems like power amplifiers [12], excavator arms [13], etc. Wiener structure, i.e., the output-nonlinear form, can describe processes like distillation columns [14], pH processes [15], biological systems [16], etc. Besides, Wiener structure can be used to deal with the dynamic differences [17,18] of transition processes at different time scales. In this paper, a simplified Wiener structure is founded as a general modeling method for describing dynamic processes with approximate monotonic nonlinearities. It should be noted that the approximate monotonic nonlinearities usually exist in process industries. For example, there is a monotonic relationship between fluid velocity and differential pressure. For another example, cold fluid temperature increases monotonically with the increase of hot fluid flow [19]. What's more, the dead-zone or saturation phenomenon [20], which is approximate monotonic, often appears in actuators or sensors.

On the other hand, for industrial nonlinear dynamic processes, the control target is to design a suitable controller, in order to make the controlled variable track the designed reference signals in the servo mode. Facing this situation, the idea of adaptive control can deal with uncertainties both inside and outside of the controlled processes [21-23], and then enhances the control performances. It means that adaptability makes output responses quickly and timely. Specifically, adaptive closed-loop systems consist of the controlled processes and the controllers. Here, the controlled processes contain actuators, real processes and sensors. Meanwhile, the adaptive controllers usually change all the time, including structures and parameters. Among these controllers, the self-tuning regulators [24] involve the changes of control parameters, and the sliding mode controllers [25-27] involve the variable structure control. Because of both complexity and diversity of the adaptive control, the related digital control algorithms are usually embedded into computer control systems (CCS) [28].

The adaptive control of continuous-time nonlinear systems has been studied extensively. The backstepping design method is generalized to the nonlinear continuous-time systems, which can be transformed into output feedback forms or parametric strict-feedback forms [29-32]. The results have also been extended into MIMO systems [33-35]. In contrast to the above results of continuous systems, their discrete counterparts remain largely unexplored, and the Lyapunov design for stability analysis of discrete models becomes much more intractable.

In seminal works [36,37], the adaptive control schemes of linear discrete-time models have been developed successively. It should be mentioned that, in the previous literature for adaptive control, the signs of control gains are required to be known as a priori knowledge. Without a priori knowledge of control directions, it is difficult to determine the updating direction of recursive parameter estimation [38-42]. To overcome the theoretical limitation, further in [43,44], the discrete Nussbaum gain is firstly proposed to present a global stable adaptive control with unknown control directions. Later, the discrete Nussbaum gain has been developed successively for the adaptive control of nonlinear discrete systems in the forms of NARMAX (nonlinear autoregressive moving average with exogenous inputs), output-feedback

innovation also includes three points: i) a simplified Wiener structure is extracted to describe such kind of processes; ii) an adaptive control scheme is exploited to guarantee the stability of control systems; iii) Both the discrete Nussbaum gain and the dead-zone factor [50] are introduced into recursive parameter updating [51–55].

The rest of the paper is organized as follows. A unified Wiener structure is analyzed for process modeling in Section 2. In Section 3, a nominal simplified Wiener structure is proposed for modeling of the dynamic processes with approximate monotonic nonlinearities. Next, the adaptive control design scheme is exploited in Section 4. Section 5 gives the stability analysis of the proposed control method. Illustrative examples are shown in Section 6. Finally, conclusions are drawn in Section 7.

## 2. A unified Wiener structure in process modeling

For lots of univariate nonlinear dynamic processes, a following unified Wiener structure (UWS) is a good choice for process modeling

$$v(t) = \sum_{i=1}^n g_i u(t-i), \quad y(t) = f(v(t)), \quad (1)$$

where  $v(t)$  denotes an intermediate variable, and  $f(\cdot)$  is a continuous function. The unified modeling based on Eq. (1) has two reasons: (a) the process dynamics can be replaced by adequate input dynamics in  $v(t)$ ; (b) Wiener structure that separates the static part from the dynamic one can describe nonlinear system characteristics sufficiently. Hence, the unified structure in Eq. (1) is able to achieve full dynamic modeling of nonlinear industrial processes. Next, several cases are listed to show universality of the structure in Eq. (1).

**Assumption 1.** Let  $C_0$  be a finite constant. For the UWS in Eq. (1), the linear dynamic block denotes a finite impulse response model, and this block is stable, i.e.,  $\sum_{i=1}^n |g_i| \leq C_0$ . That is, for arbitrary bounded inputs  $u(t)$ , the intermediate variable  $v(t)$  is also bounded. Besides, for the bounded intermediate variable  $v(t)$

Substituting Eqs. (4) and (5) into Eq. (3) gives

$$y(t) = a_1u(t - 1) + a_2u(t - 2) + a_3u(t - 3) \cdots + b_1u(t - 1)^2 + b_2u(t - 2)^2 + b_3u(t - 3)^2 + \cdots + c_1u(t - 1)u(t - 2) + c_2u(t - 1)u(t - 3) + c_3u(t - 2)u(t - 3) + \cdots \tag{6}$$

In Eq. (6),  $\{a_i, b_i, c_i\}$  are coefficients, and only the second-order terms of nonlinearity are shown. Then, the UWS in Eq. (1) are used to model the system in Eq. (2), and the nonlinear part is adopted as following

$$y(t) = f(v(t)) = v(t) + d_1v(t)^2 + d_2v(t)^3 + \cdots \tag{7}$$

Combining Eqs. (1) and (7), both linear and second-order nonlinear terms are shown as

$$y(t) = g_1u(t - 1) + g_2u(t - 2) + g_3u(t - 3) + \cdots + d_1g_1^2u(t - 1)^2 + d_1g_2^2u(t - 2)^2 + d_1g_3^2u(t - 3)^2 + \cdots + e_1u(t - 1)u(t - 2) + e_2u(t - 1)u(t - 3) + e_3u(t - 2)u(t - 3) + \cdots \tag{8}$$

Comparing Eq. (8) with Eq. (6), it is easy to see that both equations have the same structure. This means that the Wiener structure in (1) is reasonable and sufficient to model the system in Eq. (2).

**In case (ii)**, a process with the input-nonlinearity, i.e., a Hammerstein structure, is compared with the UWS in Eq. (1). Consider the following Hammerstein form

$$v(t) = u(t) + d_1u(t)^2 + d_2u(t)^3 + \cdots, \quad y(t) = \sum_{i=1}^n g_i v(t - i). \tag{9}$$

By omitting the intermediate variable  $v(t)$ , the form in Eq. (9) can be rewritten as

$$y(t) = g_1u(t - 1) + g_2u(t - 2) + g_3u(t - 3) + \cdots + g_1d_1u(t - 1)^2 + g_2d_1u(t - 2)^2 + g_3d_1u(t - 3)^2 + \cdots \tag{10}$$

Comparing Eq. (10) with Eq. (8), Eq. (10) does not contain the second-order nonlinear terms  $u(t - i)u(t - j)$ ,  $i \neq j$ . Hence, from the perspective of structure decomposition, the second-order nonlinearities in Eq. (1) are rich enough, and more nonlinear relationships are revealed between various moments of the input. Further from Eq. (8), the general expression of Eq. (1) is yielded

$$y(t) = f_W(\{u(t - i)|_{i=1}^n\}). \tag{11}$$

Define the general expression of Eq. (10) is equal to  $y(t) = f_H(\{u(t - i)|_{i=1}^n\})$ . Then, we have

$$f_H(\{u(t - i)|_{i=1}^n\}) \subseteq f_W(\{u(t - i)|_{i=1}^n\}). \tag{12}$$

From Eq. (12), it is known that the Wiener structure is a more complex structure that contains rich nonlinear properties. By selecting a suitable nonlinear function  $f_W(\cdot)$ , the structure in Eq. (1) is able to model the Hammerstein system in Eq. (9) properly.

**Remark 1.** In the block-oriented nonlinear systems, a Hammerstein-Wiener system is a nonlinear-linear-nonlinear structure. According to the above analysis, for continuous nonlinearities, the following relationships might be true

$$\{Nonlinear - Linear - Nonlinear\} \subseteq \{Linear - Nonlinear - Nonlinear\} \subseteq Wiener. \tag{13}$$

Similarly, a Wiener-Hammerstein system is a linear-nonlinear-linear structure. For continuous nonlinearities, the following relationships might be true

$$\{Linear - Nonlinear - Linear\} \subseteq \{Linear - Linear - Nonlinear\} \subseteq Wiener. \tag{14}$$

Thus, we can deduce that both Hammerstein-Wiener and Wiener-Hammerstein structures might be converted into the nominal UWS in Eq. (1). That is, for continuous nonlinearities, there might be

$$f_{H-W}(\cdot) \rightarrow f_W(\cdot) \text{ and } f_{W-H}(\cdot) \rightarrow f_W(\cdot). \tag{15}$$

From the perspective of model structure, Wiener structure is the special case of Wiener-Hammerstein and Hammerstein-Wiener structures. But in our paper, we discuss the structure transformation. From the perspective of structure transformation, univariate block-oriented nonlinear structures might be converted to Wiener structure because Wiener contains rich nonlinear relationships between various moments of the input.

**Remark 2.** Mechanism equations of industrial processes usually ignore the dynamic transition processes. To simulate the dynamic processes, the UWS in Eq. (1), which has a dynamic part followed by a static nonlinear part, can take full advantages of static modeling resources provided by mechanism analyses. Meanwhile, the above two cases illustrate that the UWS can describe many univariate nonlinear dynamic processes effectively.

### 3. A simplified Wiener structure for modeling of the nonlinear dynamic processes

In this paper, both modeling and adaptive control are considered for the dynamic processes with approximate monotonic nonlinearities.

However, if the UWS in Eq. (1) is used for modeling of these processes, the unknown intermediate variable  $v(t)$  makes system identification difficult. Thus, a simplified Wiener structure (SWS) is exploited in this paper.

Assume that a continuous monotonic function  $f_W(\cdot)$  can be used to describe approximate monotonic nonlinearities of processes. In the meantime,  $f_W(\cdot)$  has its inverse function  $g_W(\cdot)$ . The expression of  $g_W(\cdot)$  can be shown as

$$v(t) = g_W(y(t)) = \mathbf{b} \cdot \mathbf{g}^T(y(t)) = \sum_{j=1}^q b_j g_j(y(t)), \tag{16}$$

where  $b_1 = 1$ ,  $g_1(x) = x$ , and  $g_2(\cdot), \dots, g_q(\cdot)$  are known nonlinear base functions. In order to omit the term  $v(t)$ , the linear dynamic part of Wiener structure is modified as

$$A(z^{-1})v(t) = C(z^{-1})u(t) + v_0(t), \tag{17}$$

where  $A(z^{-1}) = a_0 + a_1z^{-1} + \dots + a_pz^{-p}$ , with  $a_0 = 1$ , and  $C(z^{-1}) = c_1z^{-1} + \dots + c_mz^{-m}$ . In (17),  $v_0(t)$  denotes the bounded noise. From Eqs. (16) and (17), the input-output relationship of Wiener structure can be obtained below

$$A(z^{-1}) \left( y(t) + \sum_{j=2}^q b_j g_j(y(t)) \right) = C(z^{-1})u(t) + v_0(t), \tag{18}$$

To give an adequate description of approximate monotonic nonlinearities, the nonlinear base functions  $\{g_j(\cdot)|_{j=2}^q\}$  in Eq. (18) are designed to have the following form

$$g_j(x) = \text{sigmoid}(h_j x) = 1/(1 + e^{-h_j x}), \quad j = 2, \dots, q, \tag{19}$$

where  $h_j$  denote the uniformly distributed random numbers in the range of (0, 1). Further, from Eqs. (16)-(19), the following proposed SWS can be expressed as

$$y(t) = - \sum_{i=1}^p a_i y(t-i) - \sum_{i=1}^p a_i \sum_{j=2}^q b_j g_j(y(t-i)) + \sum_{k=1}^m c_k u(t-k) + v(t), \tag{20}$$

where  $v(t)$  is the system error term, i.e.,  $v(t) = v_0(t) - \sum_{j=2}^q b_j g_j(y(t))$ . It should be noted that the SWS in Eq. (20) is used for modeling of the dynamic processes with both approximate monotonic nonlinearities and bounded noise.

**Assumption 2.** For the SWS in Eq. (20), the roots of  $A(z^{-1}) = 0$  lies in unit circular. Noise and the nonlinear coefficients satisfy  $|v_0(t)| \leq C_2$  and  $\sum_{j=2}^q |b_j| \leq C_3$ , with the finite  $C_2$  and  $C_3$ .

**Theorem 1.** *If the SWS in Eq. (20) satisfies Assumption 2, then the error  $v(t)$  is bounded and the system is BIBO stable.*

**Proof.** Since  $0 < g_j(\cdot) < 1, j = 2, \dots, q$ , we have

$$\left| \sum_{j=2}^q b_j g_j(y(t)) \right| \leq \sum_{j=2}^q (|b_j| \cdot |g_j(y(t))|) \leq \sum_{j=2}^q |b_j| \leq C_3. \tag{21}$$

From Eq. (21), it gives

$$|v(t)| = \left| v_0(t) - \sum_{j=2}^q b_j g_j(y(t)) \right| \leq |v_0(t)| + \sum_{j=2}^q |b_j| \leq C_2 + C_3 < \infty. \tag{22}$$

From Eq. (17) and Assumption 2

From Eq. (25), the nonlinear regression of  $y(t + 1)$  is given

$$y(t + 1) = \boldsymbol{\varphi}^T(t) \cdot \boldsymbol{\theta} + c_1 u(t) + v(t + 1), \tag{26}$$

where

$$\boldsymbol{\varphi}_j^T(t) = [g_j(y(t)), g_j(y(t - 1)), \dots, g_j(y(t + 1 - p))], \quad j = 1, \dots, q,$$

$$\boldsymbol{\theta}_j^T = [-a_1 b_j, -a_2 b_j, \dots, -a_p b_j], \quad j = 1, \dots, q,$$

$$\boldsymbol{\varphi}_u^T(t) = [u(t - 1), u(t - 2), \dots, u(t + 1 - m)],$$

$$\boldsymbol{\theta}_u^T = [c_2, c_3, \dots, c_m],$$

$$\boldsymbol{\varphi}^T(t) = [\boldsymbol{\varphi}_1^T(t), \boldsymbol{\varphi}_2^T(t), \dots, \boldsymbol{\varphi}_q^T(t), \boldsymbol{\varphi}_u^T(t)],$$

$$\boldsymbol{\theta}^T = [\boldsymbol{\theta}_1^T, \boldsymbol{\theta}_2^T, \dots, \boldsymbol{\theta}_q^T, \boldsymbol{\theta}_u^T].$$

Under the bounded system error  $v(t)$ , the objective is to design an adaptive control input  $u(t)$  so that the output  $y(t)$  tracks a bounded reference trajectory  $y_a(t)$ , and all the signals in the SWS are bounded. From Eq. (26), the tracking error is obtained as

$$\begin{aligned} e(t + 1) &= y(t + 1) - y_a(t + 1) \\ &= c_1 \cdot [\boldsymbol{\varphi}^T(t) \cdot \boldsymbol{\theta}_c + u(t) + c_1^{-1} v(t + 1) - c_1^{-1} y_a(t + 1)]. \end{aligned} \tag{27}$$

where  $\boldsymbol{\theta}_c = c_1^{-1} \boldsymbol{\theta}$ . It is obvious that the tracking error can be made identically zero, by choosing  $u(t)$  so that

$$u(t) = -\boldsymbol{\varphi}^T(t) \cdot \boldsymbol{\theta}_c + c_1^{-1} \cdot y_a(t + 1) - c^{-1}$$

Since adaptive control of the SWS is conducted under the bounded system errors, the idea of dead-zone factor  $l(t)$  [50] is introduced to deal with this type of errors

$$l(t) = 1, \quad \text{if } |\varepsilon(t)| > \sigma, \quad (31)$$

where  $\varepsilon(t)$  denotes the augmented tracking error. In summary, the following updating law is proposed to estimate parameters of the SWS.

$$\boldsymbol{\varphi}^T(t-1) = [\boldsymbol{\varphi}_1^T(t-1), \boldsymbol{\varphi}_2^T(t-1), \dots, \boldsymbol{\varphi}_q^T(t-1), \boldsymbol{\varphi}_u^T(t-1)], \quad (32)$$

$$G(t) = 1 + |N(p(t))| \quad (33)$$

$$\varepsilon(t) = \gamma e(t)/G(t) \quad (34)$$

$$\hat{\boldsymbol{\theta}}_c(t) = \hat{\boldsymbol{\theta}}_c(t-1) + \boldsymbol{\varphi}^T(t)$$



### 5. Tracking performance and stability analysis

**Definition 1.** Let  $q_1(t)$  and  $q_2(t)$  be two discrete scalar or vector signals,  $\forall t \in N^+$ . We denote  $q_1(t) = O[q_2(t)]$ , if there exist positive constants  $m_1, m_2$  and  $t_0$  such that  $\|q_1(t)\| \leq m_1 \max_{t' \leq t} \|q_2(t')\| + m_2, \forall t > t_0$ .

**Definition 2.** The discrete Nussbaum gain  $N(p(t))$  is proposed and defined as

$$N(p(t)) = p_s(t) \cdot s_N(p(t)), \quad p_s(t) = \sup_{t' \leq t} \{p(t')\}, \tag{40}$$

where  $p(t)$  is a discrete sequence with  $p(0) = 0$ , and  $s_N(p(t))$  is the sign function.

Then,  $s_N(p(t))$  is defined in a following manner.

The determination of the sign function at the next moment

**Begin**

1.  $s_N(p(0)) = +1$
2. **If**  $s_N(p(t_1)) = +1$
3. **If**  $\sum_{t'=0}^{t_1} (N(p(t')) \cdot \Delta p(t')) > p_s^{3/2}(t_1)$   
 $s_N(p(t_1 + 1)) = -1$
5. **Else**  
 $s_N(p(t_1 + 1)) = +1$
7. **End if**
8. **End if**
9. **If**  $s_N(p(t_1)) = -1$
10. **If**  $\sum_{t'=0}^{t_1} (N(p(t')) \cdot \Delta p(t')) < -p_s^{3/2}(t_1)$
11.  $s_N(p(t_1 + 1)) = +1$
12. **Else**
13.  $s_N(p(t_1 + 1)) = -1$
14. **End if**
15. **End if**

Let

$$Z(p(t)) = \sum_{t'=0}^t (N(p(t')) \cdot \Delta p(t')), \tag{41}$$

then the following lemmas are true.

**Lemma 1.** If  $p_s(t)$  increases without bound, then

$$\sup_{p_s(t) \geq \tau_0} \frac{1}{p_s(t)} Z(p(t)) = +\infty, \quad \inf_{p_s(t) \geq \tau_0} \frac{1}{p_s(t)} Z(p(t)) = -\infty. \tag{42}$$

**Lemma 2.** If  $p_s(t) \leq \tau_1$ , then  $|Z(p(t))| \leq \tau_2$ , where  $\tau_1, \tau_2$  are some positive constants.

**Theorem 2.** Consider the adaptive closed-loop system consisting of the proposed SWS in Eq. (20) under Assumption 2, the proposed control law Eq. (29) with recursive parameter updating laws Eqs. (32) -(39). Afterwards, the term  $G(t)$  in Eq. (32) is guaranteed to converge to some constant, i.e.,  $\lim_{t \rightarrow \infty} G(t) = h$ . The tracking error  $e(t)$  satisfies  $\lim_{t \rightarrow \infty} 0.6.3761\ 176.79\ 248.4$

Define a positive definite function  $V(t)$  as

$$V(t) = \left\| \tilde{\theta}_c(t) \right\|^2 + (\tilde{c}_I(t))^2. \tag{44}$$

where  $\tilde{\theta}_c(t) = \hat{\theta}_c(t) - \theta_c$  and  $\tilde{c}_I(t) = \hat{c}_I(t) - c_1^{-1}$ . From Eqs. (34)-(36), the difference equation of  $V(k)$  is expressed as

$$\begin{aligned} \Delta V(t) &= V(t) - V(t - 1) \\ &= \left[ \tilde{\theta}_c(t) - \tilde{\theta}_c(t - 1) \right]^T \left[ \tilde{\theta}_c(t) - \tilde{\theta}_c(t - 1) \right] + 2\tilde{\theta}_c^T(t - 1) \left[ \tilde{\theta}_c(t) - \tilde{\theta}_c(t - 1) \right] \\ &\quad + (\tilde{c}_I(t) - \tilde{c}_I(t - 1))^2 + 2\tilde{c}_I(t - 1)(\tilde{c}_I(t) - \tilde{c}_I(t - 1)) \\ &= \gamma^2 l^2(t) N^2(p(t)) \varepsilon^2(t) \left( \left\| \varphi^T(t - 1) \right\|^2 + y_a^2(t) \right) / D^2(t) \\ &\quad + 2\gamma l(t) N(p(t)) \varepsilon(t) \left( \varphi^T(t - 1) \tilde{\theta}_c(t - 1) - \tilde{c}_I(t - 1) y_a(t) \right) / D(t). \end{aligned} \tag{45}$$

Note that  $v(t)$  is bounded, i.e.,  $|v(t)| \leq C_2 + C_3$ . In addition, from Eq. (39), the following equation can be obtained

$$2/c_1 \cdot \gamma N(p(t)) l(t) v(t) \varepsilon(t) \leq l(t) \cdot |2\gamma(C_2 + C_3)/\sigma c_1| \cdot |N(p(t))| \cdot \varepsilon^2(t). \tag{46}$$

According to Eqs. (33)-(39), (43) and (46), it yields

$$\begin{aligned} \Delta V(t) &\leq \frac{\gamma^2 l^2(t) G(t) \varepsilon^2(t)}{D(t)} - \frac{2}{c_1} N(p(t)) l(t) \frac{G(t) \varepsilon(t) - \gamma v(t)}{D(t)} \varepsilon(t) \\ &\leq \gamma^2 \Delta p(t) - \frac{2}{c_1} N(p(t)) \Delta p(t) + \left| \frac{2\gamma(C_2 + C_3)}{\sigma c_1} \right| \frac{l(t) |N(p(t))| \varepsilon^2(t)}{D(t)} \\ &\leq -\frac{2}{c_1} N(p(t)) \cdot \Delta p(t) + \left( \gamma^2 + \left| \frac{2\gamma(C_2 + C_3)}{\sigma c_1} \right| \right) \cdot \Delta p(t). \end{aligned} \tag{47}$$

Taking summation of the above equation gives

$$V(t) \leq \tilde{b} \cdot \sum_{t'=0}^t N(p(t')) \Delta p(t') + \tilde{c} \cdot p(t + 1), \tag{48}$$

where  $\tilde{b} = -2/c_1$  and  $\tilde{c} = \gamma^2 + |2\gamma(C_2 + C_3)/\sigma c_1|$  are some finite constants. Suppose  $p(t)$  is unbounded. From Eq. (40), since  $p(t) \geq 0$ ,  $p_s(t)$  must increase without upper bound. Hence, there must exist a constant  $t_0$  such that

$$\begin{aligned} \Delta p(t) &\leq \tau_0 \leq p_s(t), \quad \forall t \geq t_0, \\ p(t + 1) &= p(t) + \Delta p(t) \leq 2p_s(t), \quad \forall t \geq t_0. \end{aligned} \tag{49}$$

From Eqs. (48) and (49), the following inequality is satisfied,  $\forall t \geq t_0$ ,

$$0 \leq V(t)/p_s(t) \leq \tilde{b} \cdot Z(p(t))/p_s(t) + 2\tilde{c}. \tag{50}$$

According to Lemma 1, the term  $Z(p(t))/p_s(t)$  is unbounded because  $p_s(t)$  increases without upper bound. Thus, it is concluded that Eq. (50) yields a contradiction, no matter  $\tilde{b} > 0$  or  $\tilde{b} < 0$ . As a result,  $p(t)$  is bounded, as well as  $p_s(t)$ .

According to Lemma 2, the boundedness of  $Z(p(t))$ ,  $N(p(t))$  and  $V(t)$  is also confirmed, which further implies the boundedness of  $G(t)$ ,  $\theta_c(t)$ , and  $\hat{c}_I(t)$ . Since the term  $p(t)$  is a non-decreasing non-negative sequence, the boundedness of  $p(t)$  implies

$$\Delta p(t) = l(t) G(t) \varepsilon^2(t) / D(t) \rightarrow 0. \tag{51}$$

Further, Eq. (51) leads to either  $l(t) \rightarrow 0$  or  $G(t)\varepsilon^2(t)/D(t) \rightarrow 0$ . If the latter is true, it gives  $\varepsilon(t) \rightarrow 0$ . Then, if the former is true, it gives  $|\varepsilon(t)| \leq \sigma$  from the definition of  $l(t)$ . Thus, we can always conclude  $\lim_{t \rightarrow \infty} \sup |\varepsilon(t)| \leq \sigma$ . This ensures that the  $G(t)$  will converge to some constant, i.e.,  $\lim_{t \rightarrow \infty} G(t) = \lim_{t \rightarrow \infty} (1 + |N(p(t))|) = h$ . From Eq. (34), it gives

$$\lim_{t \rightarrow \infty} \sup |e(t)| = \lim_{t \rightarrow \infty} \sup |\varepsilon(t)G(t)/\gamma| \leq h\sigma/\gamma. \tag{52}$$

Since  $e(t) = y(t) - y_a(t)$  with bounded  $y_a(t)$ , it is easy to get the boundedness of  $y(t)$ . Further, from Eq. (35), the boundedness of  $\hat{\theta}_c(t)$  implies the boundedness of  $\varphi^T(t)$ . From Eq. (29), the boundedness of  $u(t)$  can be confirmed. In a word, both inputs and outputs in the SWS are bounded. This proves Theorem 2.

**Remark 7.** The design of the proposed adaptive control scheme is to achieve exact output tracking in servo control systems. The reference signals, i.e., the set points, can change all the time. As long as the recursive parameter estimation is convergent, the tracking errors are bounded. In this section, the boundedness of tracking errors is proven in Theorem 2, and the upper bound of tracking errors is related to parameters  $\{\gamma, \sigma, h\}$ . Among these parameters, both the tuning factor  $\gamma$  and the threshold value  $\sigma$  can be selected, in order to improve tracking accuracy. The big value of  $\gamma$  also helps to expedite the updating process. In fact, the function of  $\gamma$  in adaptive controllers is similar to proportional action of PID controllers.

### 6. Numerical examples

**Example 1.** In order to test the proposed SWS and the proposed adaptive control, an input-nonlinear process is considered, i.e., a Hammerstein process. This process is designed to include both a monotonic quadratic nonlinearity of input and a linear dynamic part. The expression of input-nonlinearity is shown as below

$$\begin{aligned} & \text{if } u_{low} \leq u \leq u_{up} \\ & \quad v = u_{low} + 1/(u_{up} - u_{low}) \cdot (u - u_{low})^2 \\ & \text{else} \\ & \quad v = u \\ & \text{end} \end{aligned} \tag{53}$$

where  $u_{low} = 5$  and  $u_{up} = 27$ . After Eq. (53), the linear dynamic part can be written as

$$\bar{y}(t) + \sum_{i=1}^6 \bar{a}_i \bar{y}(t - i) = \sum_{k=1}^6 \bar{c}_k v(t - k), \tag{54}$$

where the linear parameters are set to  $\{\bar{a}_1, \bar{a}_2, \bar{a}_3, \bar{a}_4, \bar{a}_5, \bar{a}_6\} = \{0.12, 0.03, 0.05, 0.02, 0.01, 0.01\}$ , and  $\{\bar{c}_1, \bar{c}_2, \bar{c}_3, \bar{c}_4, \bar{c}_5, \bar{c}_6\} = \{0.61, 0.21, 0.06, 0.02, 0.01, 0.02\}$ . Next, the output of nonlinear dynamic system is contaminated by stochastic noise

$$y(t) = \bar{y}(t) + v_0(t). \tag{55}$$

In Eq. (55),  $v_0(t)$  denotes white noise with zero-mean and variances  $0.1^2$ . The control objective is to make the outputs track the multiple step reference signals  $y_a(t)$ . For the data length  $L = 80000$ , the expectation  $y_a(t)$  is depicted in Fig. 1.

Further, the SWS in Eq. (20) is adopted as the identification model. For Eq. (20), the linear orders are set to  $p = 10$  and  $m = 25$ , and the number  $(q - 1)$  of sigmoid base functions is set to 15.

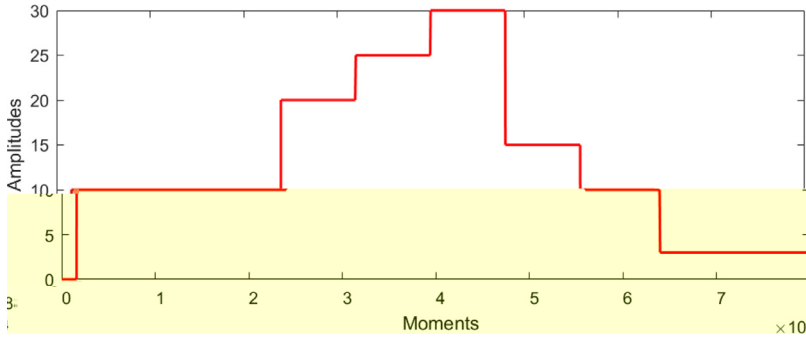


Fig. 1. The multiple step reference signals.

At this time, the recursive parameter updating laws Eqs. (32)-(39) are implemented, and the adaptive control Eq. (29) is conducted to produce the controlled inputs. In the specific algorithmic settings, there are  $\gamma = 0.5$  and  $\sigma = 0.002$ . Besides, the actual constraints for process inputs are also introduced as below.

- (i) If the input at epoch  $t$  has exceeded the range, assign the bound values to  $u(t)$ .
- (ii) If the controlled inputs are on the boundary for three times in a row, the parameter estimates are cleared to zeros, and the tuning factor  $\gamma$  is reduced by half.

For the implementation of input constraints, the following execution flow is designed.

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The actual constraints for the controlled inputs of process

**Begin**

1. After the input at epoch  $t$  is calculated in Eq. (29), extract the input range  $\{u_{low}, u_{up}\}$ .
  2. **If**  $u(t) > u_{up}$
  3.  $u(t) = u_{up}$ .  $flag = flag + 1$ .
  4. **Elseif**  $u(t) < u_{low}$
  5.  $u(t) = u_{low}$ .
  6. **End if**
  7. **If**  $flag \geq 3$
  8.  $\hat{\theta}_c(t) = \mathbf{0}$ .  $\hat{c}_I(t) = 0$ .
  9.  $flag = 0$ .  $\gamma = \gamma/2$ .
  10. **End if**
- 

Under the initial values of  $\hat{\theta}_c(0) = \mathbf{0}$  and  $\hat{c}_I(0) = 0$ , the run results of adaptive program are listed below. For the input-nonlinear process in Eqs. (53)-(55), the overall tracking performance is described in Fig. 2, and Fig. 3 shows the controlled inputs. From Fig. 2, it is easy to see that the tracking errors are small and bounded. Further, the discrete Nussbaum gains are illustrated in Fig. 4.

In addition, under the initial values of  $\hat{\theta}_c(0) = -0.1 * \mathbf{1}$  and  $\hat{c}_I(0) = -0.1$ , the run results are illustrated in Figs. 5-7. From Fig. 7, the discrete Nussbaum gains search between negative and positive directions, in order to detect the updating direction under different initial values.

It should be noted that the CPU main frequency of our computer is 2.30GHz, and that the simulation software is MATLAB R2020a. The average run time to generate the controlled input at each moment is 0.2166ms.

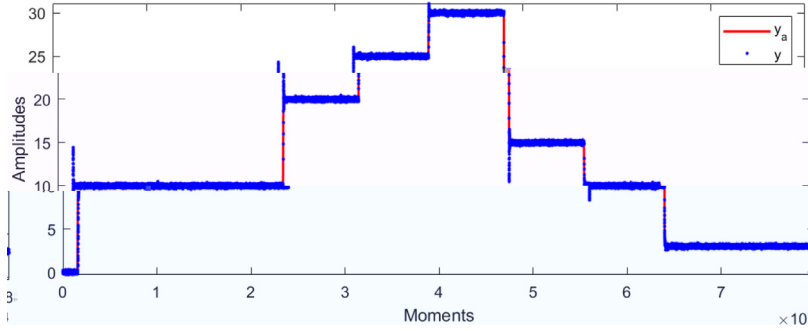


Fig. 2. Tracking performance of noisy outputs under zero initialization in Example 1.

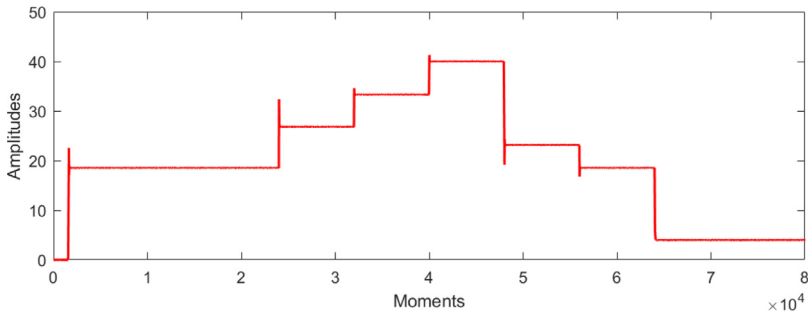


Fig. 3. Controlled inputs under zero initialization in Example 1.

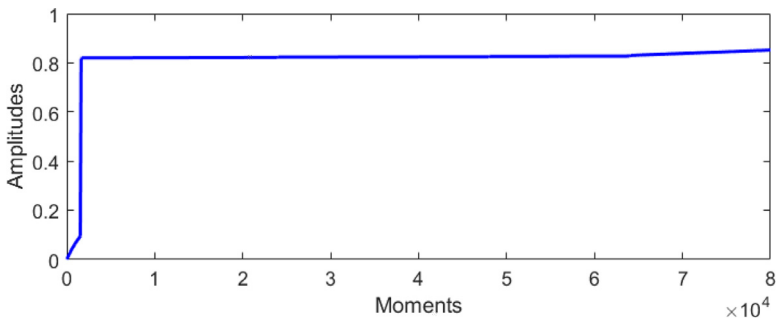


Fig. 4. Discrete Nussbaum gains under zero initialization in Example 1.

**Example 2.** In this example, an output-nonlinear process is considered, i.e., a Wiener process. This process is designed to include both a linear dynamic part and a monotonic saturation nonlinearity. The expression of dynamic part with pure time-delay is shown as below

$$v(t) + \sum_{i=1}^6 \bar{a}_i v(t - i) = \sum_{k=1}^6 \bar{c}_k u(t - m - k), \tag{56}$$

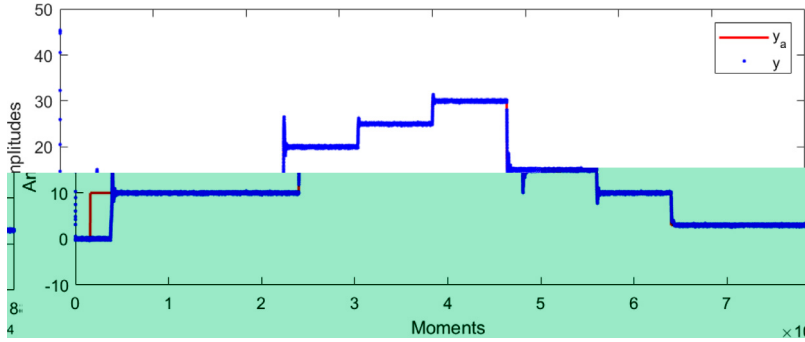


Fig. 5. Tracking performance of noisy outputs under non-zero initialization in Example 1.

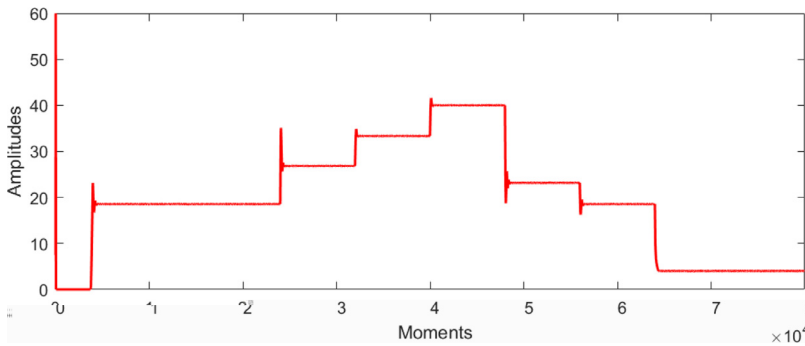


Fig. 6. Controlled inputs under non-zero initialization in Example 1.

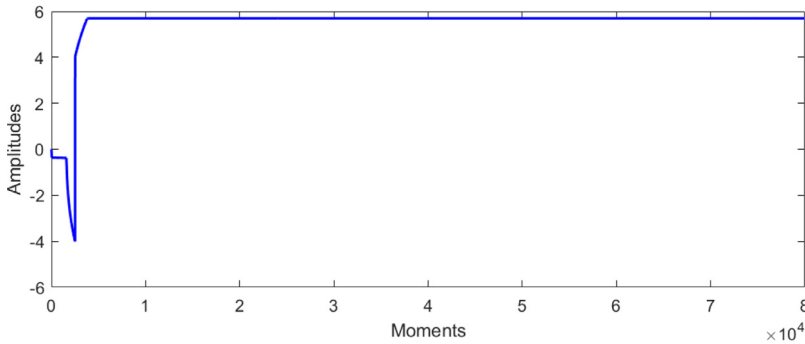


Fig. 7. Discrete Nussbaum gains under non-zero initialization in Example 1.

where the pure time-delay  $m = 6$ , and  $\{\bar{a}_i, \bar{c}_i\}$  remain the same as in Example 1. After Eq. (56), the saturation nonlinearity can be written as

$$\begin{aligned}
 & \text{if } v \geq v_{up} \\
 & \quad \bar{y} = v_{up} + \sqrt{v - v_{up}} \\
 & \text{else} \\
 & \quad \bar{y} = v \\
 & \text{end}
 \end{aligned} \tag{57}$$

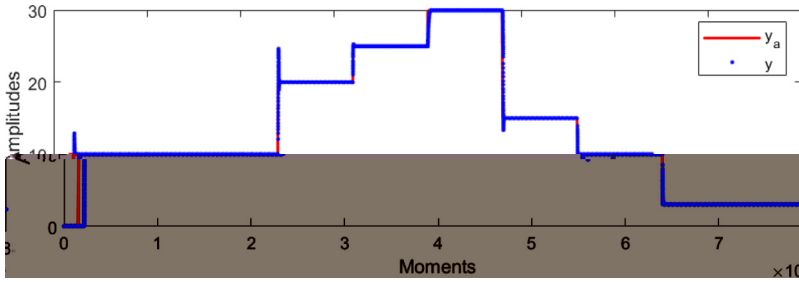


Fig. 8. Tracking performance of noisy outputs under zero initialization in Example 2.

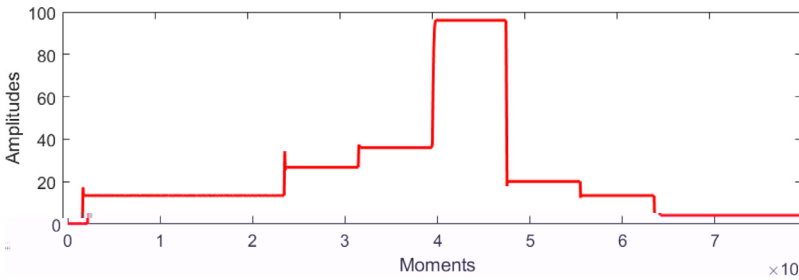


Fig. 9. Controlled inputs under zero initialization in Example 2.

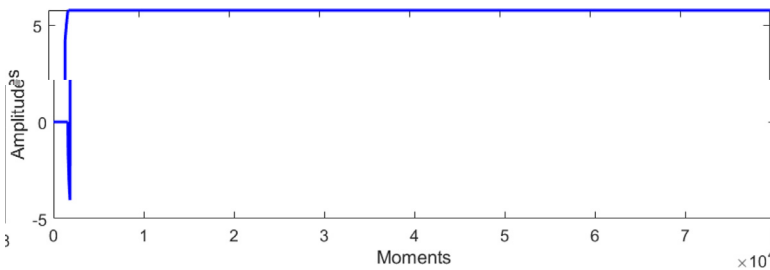


Fig. 10. Discrete Nussbaum gains under zero initialization in Example 2.

where  $\bar{y}$  denotes the noise-free output, and  $v_{up} = 23$ . Next, the output of nonlinear dynamic system is contaminated by multi-sinusoidal noise

$$y(t) = \bar{y}(t) + v_0(t), \quad v_0(t) = 0.03 \sin(\pi t) + 0.05 \sin(2\pi t) + 0.02 \cos(\pi t). \quad (58)$$

Further, the reference signals, the proposed control scheme and the specific algorithmic settings are all remain the same as in Example 1. The actual constraints for controlled inputs are also applied.

Under the initial values of  $\hat{\theta}_c(0) = \mathbf{0}$  and  $\hat{c}_I(0) = 0$ , both the proposed SWS and the proposed adaptive control are applied to produce controlled inputs. For the output-nonlinear process in Eqs. (56)-(58), the run results of adaptive program are shown in Figs. 8-10. From Fig. 8, it is known that the tracking errors are small and bounded. This also illustrates the universality of proposed adaptive control. Meanwhile, Fig. 9 shows that the controlled inputs increase dramatically when encountering the saturation effect. Besides, the average run time to generate the controlled input at each moment is 0.2011 ms.

## 7. Conclusions

Unified modeling is a challenge for nonlinear dynamic processes. In this paper, the main contributions are shown as follows: i) the proposed SWS provides a simple but effective modeling method for dynamic processes with approximate monotonic nonlinearities; ii) the proposed adaptive control method, which contains both the discrete Nussbaum gain and the dead-zone factor, provides a reliable online control scheme to achieve exact output tracking; iii) the stability of proposed adaptive control is confirmed. For specific engineering practices, both the SWS and the adaptive control can be integrated into the computer control software.

## 8. Future recommendation

However, we don't know whether the proposed SWS is appropriate for dynamic processes with non-invertible nonlinearities. In process industries, the generalization ability of Wiener structure should be further discussed. The following three aspects should be considered: i) the transformation ability of complex block-oriented nonlinear systems; ii) the descriptive ability of hysteresis or bifurcation nonlinearities; iii) the descriptive ability of non-invertible or non-monotonic nonlinearities. In view of the mentioned three points, both modeling methods and adaptive control schemes should be studied deeply, in order to achieve advanced process control.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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